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THE THOMAS YOUNG ORATION:

"SOME IMPORTANT PHYSICAL AND PHYSIOLOGICAL RELATIONSHIPS BETWEEN RADIANT ENERGY AND THE VISUAL APPARATUS AND PROCESSES"

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SYNOPSIS. Thomas Young as a scientist. Work on spherical aberration, chromatic aberration, oblique astigmatism and distortion. Modern progress along these lines, particularly as dependent upon wave length of light used. Differences between horizontal and vertical image focus. Primary and secondary astigmatic fields. Discovery of astigmatism by Young. Insufficiency of resolving power of eye. Illumination scale as a test and measure of astigmatism, both amount and axis. Accommodative mechanism. Work of Home and Ramsden. Aphakic eye accommodation. Young, Hering, and Ladd-Franklin theories of visual response. Work of Allen in support of Young's theory. Experiments of Burton and Browning. Duplicity theory. Co-ordination of retinal and cortical functions. "All or none" principle. Troland's work. Theories of vision, (a) chemical, (b) mechanical, (c) electrical. Edridge-Green theory. Houstoun's theory. Photo-ionic phenomena. Retinal current responses due to light stimulation. Quantum theory of vision.

THAT which man does lives after him. Man has in the past erected—and still continues so to erect—monuments of stone, builded by hand with great labour, and the sweat of many brows, to serve as memorials evidencing his existence. Man throughout all ages has left us vestiges of his presence and his accomplishments; from the creation day to the present hour men have left memorials, some tangible and some intangible, to be mute but eloquent reminders of the living, inward progress achieved. So our men and women of science have dreamed dreams and

have had visions; they have, by their labours, all unknowingly and unconsciously, erected temples to the glory of God as embodied in his master creation, man, and have built monuments to the perpetuation of the Truth. But these memorials have not been of stone—earthly and of the “dust to dust returneth,” but rather have they been the products of their own minds—spirit and personality, genius and skill. These men and women have not only bequeathed the work of their hands and the products of their inner selves to succeeding generations but they have also endowed those who have followed them with the spirit that actuated them. They have, by their lives and their works, perpetuated to us the true spirit of science—the Search for the Truth, which now we know but in part, for we but “see through a glass, darkly.”

We have gathered here on this occasion to pay tribute to the name of Thomas Young, and to breathe our appreciation of the love of and search for the truth which dominated this great man who lived in this great land over a century ago. True it is that Thomas Young was born an Englishman and died an Englishman. But the true scientist, while he is a member of a state, belongs to no state but rather to the world at large. For at heart the scientist—the searcher after truth and the instrument whereby the unknown may be made known—is desirous of contributing to the happiness and welfare of man some fragments which may prove good, and which may become a part of the ultimate, sum total of all knowledge, to the end that man may become master of the unseen forces about him. Hence, though Thomas Young was born an Englishman, through his contributions to his fellowmen and to succeeding generations, he became one of the world's greatest scientists and one of her first citizens. It is fit and proper that the *world* should pay tribute to the name and memory of Thomas Young. For as Dr Tscherning said in his delivery of the first Thomas Young Oration a decade and a half ago: “It is surprising that Young played the rôle of the first man in so many different branches of science. For if you take Young as the first man in the question of the theory of light, the name of the second man is Fresnel; in the question of the anomalies of refraction of the human eye, the name of the second man is Donders; in the question of colour sense, you can call the second man Clerk Maxwell, or Helmholtz; in the question of hieroglyphics the name of the second man is Champollion; in the question of terrestrial radiant heat the name of the second man is Wells, and I have not yet finished the list. For his own reputation it would certainly have been better if Young had completely developed but one of his ideas. But for the advancement of science it was better that he did as he did. For, if the second man is not frequently met with, the first one is extremely rare.”

The nineteenth century will ever be famous for the working out of the great generalisations underlying both biological and physical sciences. This vast enlargement of the boundaries of human knowledge during the past one hundred years has made it impossible for one to be first in any but a very limited field. No longer will it be possible for one to say, as did Young, that, during his last visit to Paris, he went to the Institute one Monday and had great pleasure in hearing the members of the Academy of Science discussing some theories of optics based on his own

investigations and that he went there on another day and heard the Academy of Inscriptions discussing some of his hieroglyphical discoveries, and "although it did not happen, it might very well have happened that he should have heard a discussion on his theory of life assurance in the Academy of Political Science, or a discussion of his theory of music in the Academy of Fine Arts, or even a discussion about his studies on the heart and the arteries in the Academy of Medicine. I think that the world has never seen anything like that."

But we turn from the man to his work, for of this I would speak chiefly to you, outlining some of his foundation work and the superstructure reared upon this foundation in the field of physiological optics. No less a brilliant mind than that of Tscherning was pleased to name Young as the "founder" of physiological optics. But to your present speaker he is more than the "founder"—he is the "father" and we as his followers, by reason of the spirit which possesses us, truly feel as though we were his children, as we search farther into the fields and look adown the vistas which he first pointed out and opened to our view. We shall, therefore, in that which follows, point out some of the simple but fundamental truths in physiological optics as disclosed by Young and elaborate somewhat upon the physical and physiological relationships between radiant energy and the visual processes.

First of all, let us briefly consider various researches for determining the exact nature of the image received by the human retina. Is it not true that, to be technically satisfactory, and to give an exact reproduction of both form and colour, paintings must be similar to the image received by the human retina? And is it not probable that the future will see the development of photographic lenses which will involve the important characteristics of the human eye such that, when used in connection with the three-colour process or some further development along this line, there may be a substantial reproduction of the retinal picture? For the vast preponderance of the visual world is, at any moment, represented in consciousness by ill-defined and physically inaccurate images. Hence, it would be wellnigh insolent to assume their uselessness. Rather must they be regarded as the necessary background through which the foveal image acquires a significance it could not otherwise possess. Sensations from the periphery of the retina, with all their obscurity of form and texture, must form an essential fund of knowledge. And what are these fundamental characteristics, or imperfections, if such you desire to call them? I am sure that you will agree that they may be included under the expressions, spherical aberration, chromatic aberration, oblique astigmatism and distortion.

The eye is not an achromatic combination although in everyday life the chromatic aberration is not noticeable as such. A very simple experiment showing that it does possess this aberration consists in throwing, by means of a projection lantern, reading matter or pictures made up of different colours, upon a distant screen, and noticing whether or not different coloured letters or objects appear to lie in the same plane. To nearly every observer there appears to be a noticeable depth difference. Young made the first estimates upon the chromatic aberration of the eye, while Fraunhofer made the first reliable measurements on the difference in the focal

length of the eye for extreme spectral colours. He observed a prismatic spectrum through an achromatic telescope, the eyepiece of which carried a cross-hair. Fraunhofer noticed that he had to move the ocular nearer the cross-hairs for clear vision when observing the violet region in contradistinction to the red region. Young estimated the chromatic aberration to be 1.3 D, Fraunhofer found it to be 1.5 to 3 D, and Helmholtz gave 1.8 D. One method of determining quantitatively the axial chromatism of a lens is to find the difference in inclination of rays of varying wave length which come to the same focus. In 1914 Nutting reported his investigations on the matter of the chromatic aberration of the eye in the *Proceedings of the Royal Society*. The results obtained indicate that the human eye approaches very closely in its chromatic aberration to an eye of distilled water. Taking the refractive indices of water for red (C) and for violet (G) and applying methods for calculating reduced eyes we find a chromatic aberration of 0.434 mm. which corresponds to practically 1.1 D. In Nutting's work the results on several observers, showing the relationship between relative focal distances or the axial focal lengths of the eyes and different wave lengths of light, indicated a considerable flattening through the central portion, i.e. on both sides of the D line, and the assumption was made that this is due to the eyes being more or less corrected. A similar curve for water shows no such flattening but rather a convexity toward the axis of wave length. Ames and Proctor, working at Dartmouth College, have this year published results upon the spherical and chromatic aberrations of the eye. By a very much more elaborate and exacting set-up of apparatus—which it has been the pleasure and good fortune of the writer to see and examine—they have determined the axial chromatic aberration and failed to find any flattening of the focal length—wave length curve as found by Nutting, the suggestion being made that the flattening found by Nutting was due to his using a chromatic, i.e. white light, accommodation object. Hence Ames and Proctor used monochromatic fixation; in general λ 6560. They found for the amount of axial chromatic aberration between λ 6563 and λ 4861 values varying from 0.72 D to 0.98 D. The determination of the axial chromatism of different zones is still another problem best considered in conjunction with variations of spherical aberration with wave length and zone of the eye examined.

Let us pass on to consider spherical aberration—a spreading of rays along the axis near the image plane due to different zones of the lens not having the same focal length. The great savant, Young, prepared a series of experiments which conclusively prove that the eye is not aplanatic. For a myopic eye, or one which is mechanically made so by the addition of convex lens power, sees a distant luminous point as a circle of diffusion with its brightness concentrated at the centre if the eye has positive spherical aberration; if the aberration is over-corrected or if the luminous point is inside of the far-point, the peripheral regions will be more luminous. Young gives us the experiment of passing a needle in front of the eye made myopic during the performance of this test, when the shadow of the needle will be seen in the circle of the diffusion. If the shadow remains straight at all points, there is then no appreciable or perceptible aberration; if it is concave toward the periphery ordinary aberration is indicated, but if it is concave toward the centre,

there is evidence of an over-corrected aberration. Furthermore, Young said that such experiments showed that the aberration is not always the same in different meridians.

And then too, we have the optometer of Thomas Young—of which Tscherning says: "It appears to me to be one of the most important instruments for the study of physiological optics." This instrument enables us to measure spherical aberration directly. The instrument consists of a little rule carrying a fine white line on a black background. The observer looks along this line through a +10 D lens. In front of the lens is placed a ruler free to move and carrying small slits differently spaced, so that various zones of the eyes may be examined. These slits act like the openings in Scheiner's method of testing a lens. Young determined the difference between the far-points of the central portion of the eye and the peripheral parts through the use of four slits properly placed, obtaining the aberration as the difference between the two readings just cited. We shall pass over with a mere mention such classical work as the aberroscope of Tscherning, the researches of Stadfeldt and the retinoscopic methods of investigating spherical aberration, and come to the latest work of Ames and Proctor, who have used an elaborate and exact means of passing a beam of light of very small cross-section through different zones of the eye and determining the inclination of the beams when their images coincided with the image formed by the axial ray. These experimenters found that the relationship between zone distance from the axis and the calculated distances behind or in front of the retina where incident parallel rays cut the axis in any instance and for each of the three wave lengths, red (6560), yellow (5890) and blue (4860), is, roughly speaking, of the form of a parabola with the axis line of the curve lying at about 1.5 mm. from the visual axis of the eye. And, generally speaking, their results indicate that for wave lengths of 5890 or greater the light is focussed by the different zones in front of the retina, while for the shorter wave lengths it is focussed behind. Furthermore, these investigators have examined the spherical aberration with different wave lengths of monochromatic illumination in both horizontal and vertical meridians. The results showing the spherical aberration of different zones with different wave lengths of light indicate that the aberration curves are practically independent of wave lengths used, that the forms of the curves for horizontal and vertical meridians are practically the same, but that there is for each and every wave length a difference between the focussing point of horizontal and vertical meridians. They found, for instance, in the case of one observer, that this difference of horizontal and vertical meridian image focus amounts to about 8/100 of a millimetre for λ 6563, λ 5893 and λ 4861. There is also the much mooted question as to whether the eye so focusses as to allow that part of the cone of light which has the smallest diameter to fall upon the retina or rather that portion where the section of the caustic due to spherical aberration is smallest. Tscherning and Gullstrand believe it to be that portion of the bundle where the caustic is the smallest and not where the cone has the least diameter. Ames and Proctor believe that they can definitely assert that, in the eyes examined, they focus on that portion of the bundle where the cone has the least diameter.

For a long time oblique astigmatism has been known to exist in the human eye and was first dealt with in a fairly comprehensive manner by Young. Such an astigmatism causes the rays of light from a point source not on the axis to focus into a ray bundle of quite complex character. In its pure form in a simple lens, a ray bundle from a point source focusses first to a line which lies in a tangential position to a circle about the axis of the lens; it then crosses with a narrowing in its long dimension and lengthens in the short one until it again becomes a line perpendicular to the first line, or forms what we speak of as the radial line. The farther the source from the axis of the line the greater the separation between the two parts of the image which have the forms of lines. Young, in 1801, remarked on the small extent of perfect vision. Imperfections which begin a degree or two from the axis are due, he claimed, in part to the aberration of oblique rays and again to the insensibility of the retina. He calculated the positions of the primary and secondary astigmatic image fields of a circle ten inches from the eye. He gave curves showing first and second astigmatic image fields formed by the cornea only, by the cornea and anterior surface of the lens, and finally by the cornea and the whole lens. From the time of Young onwards, we find three methods employed in the endeavour to determine the position of the primary and secondary astigmatic fields: (1) ophthalmoscopic—Stammerhaus, Schoen, Rimpler, and Staub; (2) skiascopic—Parent, and Druault, and (3) subjective determination of position of sharp focus of paraxial stimuli—Peschel, Albin, Wertheim, Heinrich, and Loria. In a general way, the results obtained by different experimental methods agree that the image of oblique rays formed by the dioptric system of the eye is astigmatic; that the retina lies approximately half-way between the primary and secondary image fields formed by a surface in space, all points of which are equidistant from the eye, and that monocular space perception is tridimensional. Ames and Proctor, in the work already referred to, have endeavoured to make accurate determinations of the position of the astigmatic fields for light of different wave length. The curves for different wave lengths show a marked similarity, but the form of the curve for the primary differs considerably from that of the secondary astigmatic field. Corresponding to the different coloured image fields there are, of course, object views in space. These can be accurately determined for any given accommodation of the eye. For example, the nature of the image of a white light point source situated two metres away being considered, it is found that the red light will be stretched in a tangential direction, the yellow light in a radial direction, and, being beyond the second astigmatic field for green and blue, these colours will be in the form of a radial diffusion circle. Hence, the two very important conclusions, (1) with any given accommodation, every object in space has its characteristic retinal image form due to its position in space relative to the fixation point, and (2) this characteristic image form is governed both by the shape and colour of the object. Thus the notion of unocular tridimensional space perception, first recognised as a possibility by Heinrich, is given a very definite basis. And again, it follows that, with any given accommodation, the nature of the images on the retina from stimuli of different wave length all at the same distance from the eye would be very different both in

shape and intensity, hence it is possible that past measurements on peripheral colour sensitivity may be somewhat invalidated. And thus we see, by way of a general conclusion at this point, the importance of researches for determining the exact nature of the image received by the human eye, with reference to which the initial investigations were carried out by Thomas Young, and, in addition, we see the intimate relationships existing between such experimentation and the technique of the art of actually reproducing that which the eye views.

In the mind of the clinician and those interested in the applications of physiological optics to the care of the human eye, the name of Thomas Young will doubtless always be associated with the discovery of astigmatism, published in the *Philosophical Transactions* for 1793. First of all, Young proved the defect in his own eye by means of his optometer and by observing the forms of the circles of diffusion produced by a luminous point. We know that he had an astigmatism of about 1.7 D against the rule. Furthermore, he proved that his astigmatism was not located in the cornea because, by performing the very celebrated experiment of putting the eye under water and substituting a spherical lens for the cornea, he found that the astigmatism persisted and to the same amount. In 1827 the astronomer Airy corrected his defective sight by means of cylindrical lenses. From that day to this there have been many methods and instruments devised for the detection of the amount and the axis of the astigmatism of the eye. The ophthalmometer is accredited in matters of discovery to von Helmholtz and was made a valuable clinical instrument by Javal and Schiötz. Helmholtz introduced into ophthalmometry the principle of doubling; it seems, however, that the method had already been used for the same purpose by Thomas Young—again showing the versatility of the mind of this genius.

An eye which has an insufficient resolving power shows, in general, three defects:

- (1) a marked loss of power to sustain the adjustments needed for clear seeing,
- (2) an undue lag in making the adjustment necessary for clear seeing, and
- (3) the fact of increased amount of light required initially to discriminate details in the standard acuity object.

In the past we have quite commonly pursued methods of examination of ocular defects involving changes in visual angle rather than illumination. But, in so far as the test object is concerned, clearness of seeing depends upon both the visual angle subtended and the illumination. Hence, either the visual angle scale or the illumination scale may be used for the detection of the defects of the eye considered as an optical system, i.e. the visual angle may be varied and the illumination kept constant or the converse. The illumination scale, however, possesses the greater sensibility for the detection of small errors, and from recent experimentation we are satisfied that visual radiant energy, i.e. illumination, constitutes the superior method in detecting the exact amount—and, in the case of astigmatism, the placement of the correcting cylinder—of visual interference which exists because of physical defects of the eye as an instrument. For if the eye has an equal resolving power in all meridians, the amount of light required just to discriminate a test object in all meridians will be the same; if the resolving power is not equal, the amount of light will be different in the various meridians and different by an amount proportional to the amplification represented by the

illumination scale. In particular is this true in the determination of the amount and the exact placement of the correction in cases of low astigmatisms. Clinical methods as ordinarily pursued may vary easily in conclusions from 0.12 to 0.25 D in strength of cylinders and, in the case of low astigmatism, from 5 to 20 degrees in the placement of the cylinder axis. These inaccuracies may affect keen acuity, speed in the use of the eye, speed of discrimination and in the adjustments of the eye for clear seeing at different distances. And the resolving power is of importance in expressing the difference in the amount of light that is required by different people as a working minimum and is of particular significance in explanations of acuity in twilight conditions and doubtless also in "night-blindness," so called. Experimental tests made by several workers—and these results have been largely verifications of the work of Ferree and Rand—show the superiority of the illumination scale for the detection of the exact amount and placement of the cylindrical corrections. To indicate the sensitiveness of this method let us cite a condition in which, with + 0.12 cyl. ax. 55°, it was found that equal illumination was required for the discrimination of the test object in all meridians. Clinical methods, as ordinarily pursued, gave + 0.25 cyl. ax. 70°. The difference in the amount of light required for the discrimination of the test object in the least favourable meridian for the two corrections was 0.39 m.c. or approximately 244 per cent. From a study of these illumination scale methods and from a first-hand acquaintance we feel that greater accuracy will be forthcoming in the future in the detection and correction of errors of refraction, and especially in the application of illumination methods to astigmatism, the error of the eye first discovered and measured by Thomas Young approximately a century and a quarter ago.

The second great merit in physiological optics attributable to Thomas Young is the first definite proof that accommodation is due to an increase of convexity of the crystalline lens. Young's paper on *The Mechanism of the Eye*, published in 1801, was written especially to support this contention. He eliminated possible corneal curvature changes during accommodation by observing that during this act there was no change in corneal images, but that an easily visible change could be produced by exerting a pressure on a peripheral portion of the cornea. This change of curvature was considerably less than that which would be necessary to explain accommodation. And Young added further to the evidence on this subject by performing his classical experiment of "putting the eye under water." This he did by taking the objective of a microscope which had as nearly as possible the same refractive power as the cornea, filling the tube with water and placing it in front of his eye, also submerged in water. The dioptric effect of the cornea was thus eliminated and was replaced by the unchangeable objective. In this experiment the amplitude of accommodation (about 10 diopters for Young) was preserved, showing that it was not a function of the cornea. Young found that accommodation is not produced by an elongation of the globe. By use of his optometer he demonstrated that persons operated on for cataract lost their accommodation power. He thus firmly established his belief that this power resides in the crystalline lens and hence that it could result only from a change in the curvature. To be sure Young incorrectly

supposed that the changes in the shape of the lens were caused by the character of the lens fibres. The manner in which the changes were produced in curvature of the crystalline lens remained unknown for a half century after Young and until the work of Bowman and of Bruecke.

It is of interest to us to note that the experiments of Home and Ramsden caused Young to abandon his initial conception of the *modus operandi* of accommodation. Home and Ramsden conceived of the act of accommodation as being due to a change in the length of the eye as well as in the curvature of the cornea. That which apparently caused Young to lose faith in his own notions was the statement that the eye of a man in which the lens had been couched was still susceptible of this adjustment. As we have said, he was led to reassert his theory of lenticular action by reason of his experiment of putting the eye under water and of discovering that several young subjects operated on for cataract had completely lost the power of accommodation. But, while rare, there are a considerable number of cases on record of the ability of an aphakic eye to read small print with the same glass as is used to see clearly at a distance. Three or four such cases have been officially reported in America within the past year. Various causes may be operative to explain this phenomenon; Jackson in an editorial in the *American Journal of Ophthalmology* for April, 1921, cites and discusses eight possibilities. But in none of the cases reported has there been any data recorded, in so far as we know, of actual observations made as to the mechanism of the altered adjustments of the aphakic eye reading through distance corrections. It is possible that considerable very valuable information will accrue from such observations.

Young also made observations which, as he said, amounted to a mathematical demonstration of his solution of the seat of the accommodative changes, these observations showing that the accommodation is much stronger in the centre of the pupil than near the margin; a circumstance which he believed proved that the surfaces of the lens flatten near the margin at the same time as the curvature increases in the centre. We are all aware of the very elaborate and exhaustive experiments which corroborated and extended the early work of Young and which were carried out by none other than the greatest of living physiological opticians, Tscherning. True it is that we have the two visual theories of Helmholtz and Tscherning with numerous adherents to both theories. We may, as we pass on to other topics, well pause long enough to wish that we may be fortunate enough to possess in our day and age a complete and irrefutable theory of the mechanism of accommodation. Possibly we already possess it and fail in our ability to apply it in all particulars.

We turn from a consideration of some of the physical relationships between radiant energy and the eye as a physical instrument, in which Thomas Young played so important a part as the original investigator, to the presentation of some of the physiological and physical relationships between radiant energy and the visual processes. We have immediately before us such important problems as those of explaining the phenomenon of vision, of deciphering the enigma of colour vision, and of settling the question as to whether it is the character of the radiant energy or

the quantity of the same which may produce marked retinal changes under extreme conditions and which may be one of the primal causes, if not the primary cause, of definite changes in the structure of the ocular media, especially the crystalline lens. Far be it from me to presume to answer definitely any of these most important and, as yet, unsolved problems in the field of physiological optics. But we may, with great propriety, discuss colour vision, for Thomas Young gave the world the first reasonably satisfactory theory of colour vision which, as modified by Helmholtz and bearing the name of the Young-Helmholtz theory, has received the approval of many scientific minds. And it is indeed fitting to discuss in this oration the nature of the visual process or processes, for we may well believe that Thomas Young, were he living to-day, would endeavour to fathom the actual mechanism of vision, which would include not only processes occurring in the eye but also those which are located in the optic nerve and the brain centres to which the conductors of this nerve lead. For I think we may rest assured that the *retinal* factors in the process have been over-emphasised, for the visual consciousness must depend fundamentally upon central nervous processes and only indirectly upon those of the retina. For we must needs consider the correlation of the retinal factors with the essential nervous system. Due to the excellent work of Lucas, Adrian, Nernst, and others on the physical basis of nerve excitation, including much information as to the nature of the nerve fibres and synaptic activities, we are now possessed of knowledge of nerve functions. And the investigations of Sherrington, Head and others throw a great deal of light on the nature of the synaptic or nerve processes.

There are extant some sixty or more theories of the mechanism of the visual response. Three have proved sufficiently valuable to be discussed in textbooks on physics, physiology and psychology. These three are those of Young (and Helmholtz), Hering, and Ladd-Franklin. All of these theories postulate the existence in the retina of a number of specific chemical substances which are acted upon by light. There are, according to the theory of Young, three of these substances which are decomposed at maximum rates by red, green, and blue (violet) light respectively. These decompositions are reported to the brain along the path of the optic nerve. Young laid down the premise that white light is composed of a mixture of three colours only, namely, red, green, and violet, and postulated the existence of nerve fibres in the retina composed of these sub-members or fibres, each provided with a special terminal organ. Helmholtz modified this hypothesis by assuming that each spectral colour irritated all these fibres at once but in a different degree. Koenig, Maxwell, Abney, and others, by studying the colour sensations of normal eyes as well as of colour-blind persons, have drawn three curves showing the sensitiveness of the assumed three primary sets of nerves to stimulation by light of different wave lengths. The theory of Young serves to explain many facts of colour vision, but it appears to fail completely to explain the radical changes in the character of the colours which are experienced when certain ratios of the elementary processes are realised: for example, the combination of red and green to form yellow or of yellow and blue to form white. And the phenomena of after-images are only approximately reconcilable with this theory and the problem of simultaneous contrast is not

satisfactorily explained. However, the three colour curves receive independent verification along different lines, notably from the distribution of hue sensibility in the spectrum, which coincides with the resultant of the slopes of the chromatic curves at any wave length, and again from the facts of pure chromatic minuthesis, serving to identify the intersection points of the curves.

The recent work of Allen, resulting in conclusions in support of the Young theory, is particularly impressive. In Allen's experimentation a long series of persistency curves was made under the following conditions. A certain colour in an arc light spectrum was isolated and the eye kept constantly fatigued with it. With the retina in this condition, persistency measurements were made throughout the second spectrum. The persistency curves thus obtained were compared with those procured with an eye in a normal condition. By means of his work Allen believes that he has established the fundamental red sensation as lying between the red and the point of transition at $\lambda 6600$. Similarly, the fundamental green sensation is excited by some portion of the spectrum lying between the second and third transition points at wave lengths $\lambda 5700$ and $\lambda 4700$. And the third fundamental sensation, which he says must be violet and not blue, lies between the fourth point of transition at $\lambda 4200$ and the end of the violet. Proceeding through the spectrum, we pass from the fundamental red, through the transition point at $\lambda 6600$ to compound sensations (yellow), through the transition point $\lambda 5700$ to the fundamental green sensation, through the transition point $\lambda 4700$ to compound sensations (blue), and through the transition point $\lambda 4200$ to the fundamental violet sensation.

Barton and Browning have investigated mathematically and with the aid of models the probable mechanism of the Young-Helmholtz theory. Three resonators of appropriate natural vibration periods were subjected to forced vibrations. Photography was employed to record their respective amplitudes of response; the natural periods corresponded to $\lambda 7600$, 5500 and 4000 . Mathematical analysis showed that the vibrations must be very highly damped in order to produce the resonance curves required by the facts of colour vision. The damping demanded is so great, however, that the persistence of vision cannot be explained as a result of the continued action of the vibrations after cessation of the stimulus. The ratios of the responses of the three systems of vibrators to different impressed frequencies were found experimentally to fit in very well with many facts of colour vision: such, for example, as the mixing of various monochromatic portions of spectral energy.

The theory of Hering also postulates the existence of three substances; one of these substances is decomposed by lights of all wave lengths, although to the greatest extent by yellow-green light. It builds itself up in the absence of light. The other two substances are decomposed at a maximum rate by red and yellow lights respectively and are recomposed at maximum rates by green and blue lights respectively. The six psychological primary colours are correlated with the six different and distinct rates of change occurring in the three substances. We have here a satisfactory explanation of the manner in which these primaries combine with one another, accounting for the antagonistic behaviour of red and green, and yellow and blue. Negative and complementary after-image effects are also explained

and the facts of colour vision doubtless as satisfactorily presented as in the Young-Helmholtz theory. Verworn and others have criticised the Hering theory in the matter of the assumption that anabolic processes can be brought about by the direct action of a stimulus. Grave objections to the Hering theory lie in the failure of opposite processes in the black-white substance to cancel each other, while those in the other two substances always leave a residual gray, and the fact that psychologically primary red and green do not combine to give a gray but instead a yellow. And last of all we should mention the failure of continuous stimulation of a single retinal region to reduce the effects (sensory) of all stimuli to a neutral mid-gray. The first difficulty has been discussed by Mueller, who differentiates between retinal and brain centre processes, placing the substances postulated by Hering in the retina and assuming the existence of an additional gray-producing process in the centres. The third difficulty mentioned would seem to be overcome only by giving up the cardinal feature of the Hering theory, i.e. the antagonistic natures of the processes involved in the three pairs of primary colours. Some few years ago Troland propounded a theory obviating this difficulty, in which he suggested five distinct light-sensitive substances in the retina, each acted upon by light, the rate of decomposition being at a maximum at different wave lengths for the different substances. One substance was responsible for white light and the other four for red, yellow, green and blue respectively. Blue is represented as an unstimulated condition of the visual brain centre.

The Ladd-Franklin theory is possibly the most satisfactory of all the classical theories, as many call them, in that its basic points are probably essential to a satisfactory elucidation of the visual process. In her theory a single light-sensitive substance assumedly exists in the retina. This substance exists in three stages of differentiation. In the first stage its decomposition leads to various gradations of achromatic luminosity ranging from black, in which there is an absence of stimulus, to white, in which there is a maximum rate of decomposition. In the second stage of change of substance we find postulated two parts, one of which is decomposed at the maximum rate by yellow light and the second part by blue light, the products of these decompositions respectively acting upon or being transmitted by the optic nerve to produce yellow or blue, as the case may be, in consciousness. Simultaneous and equivalent decomposition of the two parts of the substance again gives the original or first stage of stimulation and thereby gives a gray. The third stage of decomposition or change of substance consists in the breaking up of the yellow-sensitive component into red and green-sensitive constituents respectively. We see, therefore, that red and green are antagonists, as in the Hering theory, but the product is yellow instead of white. The yellow, coming into antagonism with blue, gives us the quality of white or gray.

But there are two distinct mechanisms of response in the retina; the cones—dominant in day vision and giving both achromatic and chromatic sensations, and the rods—giving responses to low illumination or furnishing twilight vision and giving only achromatic sensations. We feel that this *duplicity theory*, which has to do with the differentiation of functions only, is reasonably well established. All

things considered, it seems that the Ladd-Franklin theory—accounting for the psycho-physical response of the retinal cones and taking into account the three-colour mixture laws upon which the Young-Helmholtz theory is based—supplemented by the assumption of an independent red response for twilight vision, constitutes one of the best working (if not the best) hypothesis we possess of what we may call the retinal processes in contradistinction to that which may occur from the retina back to and including the brain centres.

The more primitive the theory of vision the more certain is the visual consciousness considered as being determined by the object. The most advanced ideas or conceptions look upon the visual projection areas of the cerebral cortex as of prime importance. The facts of pathology clearly indicate that we are justified in postulating in the cerebrum the existence of highly elaborated and differentiated visual mechanisms and processes. There is apparently a special cerebral responding centre for every distinct feature of the optic function. And pathological changes in the visual areas of the cerebral cortex are capable of causing pronounced changes in colour vision without any equivalent effect upon other visual functions. We do not, for instance, need any assumption of retinal contrast mechanism, because the function of contrast can remain operative when lesions in the projection areas have practically annulled intercourse between the higher cerebral areas and the retina. Modifications in colour vision due to pathological conditions in the visual cortex indicate the existence of definite cortical processes corresponding to the constituents of the visual consciousness.

In all three of the prominent theories—Young, Hering, and Ladd-Franklin—there is the temptation, at least, to correlate the cerebral reactions with individual nerve centres; thus demanding a red-perceiving centre, for example, as the recipient of red retinal process product. Recent investigations rather lean toward the conclusion that specific qualities of sensation depend more upon characteristic nerve processes than upon anatomic centres. In nerve physiology we know that certain reflexes antagonise and inhibit each other; it is apparent that such a mutual inhibition exists between the cerebral components for red and green respectively. But this antagonism which inhibits both red and green, for instance, does not kill out the energies involved, but generates the yellow component. And in a similar fashion yellow and blue inhibit each other but thereby produce the sensation of white.

Certainly the scheming out of a probable cerebral mechanism of colour vision is a Herculean task. From the retina to the cortical area, we may conceive of receiving stations for the various functions. The basic functions are those of brightness, form and colour. Disorders in the calcarine region of the occipital cortex attack the colour function first of all, then the perception of form and, finally, the recognition of brightness. It is likely that the chromatic function is centred in one of the higher strata of the cortical mechanism. Should we proceed backwards from the cortical area ultimately involved toward the retina, we may well pause to ask how the several factors in the visual processes are represented in the nerve at different neural levels. The arguments in support of the Young-Helmholtz theory indicate that, between the cortical region concerned and the retina, the nerve conduction must involve

three independent variables. Helmholtz assumed that there were three different kinds of nerve fibres and receptors in the retino-neural mechanism. Others, equally philosophical, have postulated a threefold qualitative differentiation of the nerve currents passing along a single nerve fibre. This latter hypothesis seems, from recent experimentation, to be more difficult of acceptance than at the time of its conception. For, from the researches of Adrian and Lucas and the demonstration of the so-called "all or none" principle for the response of nerve tissue, we find that an impulse which passes along a nerve fibre is incapable of *quantitative* variation, being fixed in magnitude by the properties of the fibre. The number of nerve fibres in simultaneous action would afford gradation in muscular activity. Or the basis of gradation might be the frequency of the impulses, i.e. whether they are densely or sparsely distributed. In visual sensations we can hardly explain variations in intensity or luminosity on the number of fibres in action, since the number of fibres presumably explains the variation in the area of retina stimulated. Evidently, then, it is possible to explain variations in intensity of stimulation of the retina by means of the conception of nerve impulse-frequency. Recently Troland, employing a well-known entoptic phenomenon first described by Purkinje, has measured the brightness of the effect produced by the stimulation of certain retinal receptors by the nerve currents generated in the conducting fibres of the retina as a result of stimulation of other receptors, and has found that this brightness is independent of the intensity of the stimulus which is initially applied to the retina. Apparently the frequency is the only factor which can explain luminosity variations. If this is true, then theories of vision based on the analogy of the telephone cannot be accepted, since variations in amplitude of the nerve pulses to transmit the intensity cannot be admitted to be true according to the "all or none" principle. And further, the telephone idea of visual response cannot be admitted because of the fact that a telephonic representation of a light wave in a nerve current should be composed of waves whose lengths are smaller than the diameter of an electron. If the visual currents are actually quantal in character, they must have frequency and this frequency will be variable. Such a variation may easily be presumed to form a basis for the transmission to the brain of the intensity of luminosity, for brightness vision precedes form and colour vision. The idea of impulse frequency as the basis of brightness vision satisfactorily explains achromatic vision, but fails to afford a basis of colour perception. But hue and saturation enter in a very minor way into laws which govern luminosity, and suggest the physiological distinctness of mechanism underlying chromatic and achromatic vision. Furthermore, that luminosity and chromatic values are actually independent factors in the physiological processes fundamental to vision is indicated by the decided differences in the proportionality constants relating these two factors for different spectral colours. The luminosity may be merely a proportional accompaniment of the chromatic value of the sensation. Recent work by Ferree and Rand indicates that the periphery of the retina—in which rod vision predominates—is not blind to red, blue and yellow, for with stimuli of sufficient intensity the limits of red, blue and yellow coincide with the limits of white light vision; the limits for green, however, seem to be

absolute. But the amount of change of intensity required to produce a detectable change in the apparent limits of sensitivity in the more remote parts of the retina is very large. The implication that luminosity and colour vision are not intimately related, but are quite largely independent, is evident.

If we consider the chromatic function by itself, the evidence that it depends upon three selectively sensitive mechanisms is apparently very strong. The individual cones may be trichromatic or there may be three separately connected and distinctive species of receptors underlying colour vision. Deliverances of responses of adjacent elements would undergo fusion in the brain; otherwise we should never see "white" as such, but as mosaics of three colours. Recently Hartridge has demonstrated that such fusion does occur, for minute colour patterns falling upon a considerable number of cones are perceived in their combined or integrated effect. If the retinal cones are differentiated into three classes, then it is possible that the "all or none" principle should hold, and that the three species of conducting fibres should differ functionally, for the individual impulses travelling along them might have different characteristic amplitudes. Troland finds that there are significant differences in the thresholds for different colours used to produce the "blue arc" effect.

And again, it is possible that phase, or phase difference, may be present in the propagations of impulses along nerve tracks of the visual mechanism. Such is true in audition; but localisation in vision does not rest upon differences of phase as in the localisation of sound. With a uniform luminosity, colours might be due to particular relative displacements of the phases of adjacent impulse streams. Achromatic sensation would depend upon exact synchronism of adjacent impulses, while chromatic vision would depend upon definite deviations from this condition. We can easily conceive of phase differences as being produced by differences in the exact position of the source of the impulses in the retinal layer. Experiments recorded by Koenig and Zumft indicate that the retinal layers which are sensitive to the various parts of the spectrum do not coincide. This difference in position of sensitivity might be due to the relative positions of specific responsive substances within the receptor organs or to other properties of the same. We may comment that the laminae into which the rods and cones split up transversely to their lengths are admirably adjusted to produce optical interference effects within the receptors. But the duration and rate of propagation of the nerve pulses are large as compared with the dimensions of the rods and cones, and this throws grave doubt on such a possibility as that mentioned. But Darzens used the principle of interference in a somewhat different manner, for he supposed standing waves produced by reflection between the receptors, and that the energy nodes and loops are situated at different distances and positions for various wave lengths.

At any rate, the effect of the sensations of light in the retina is perceived by the brain as a visual sensation. The process or processes are still very obscure. Some of these we have hinted or have briefly discussed; we have said but little, however, as to the possible means whereby an ether disturbance can produce a reaction ultimately recognised by the brain as a visual sensation. Broadly speaking,

we have three theories: (a) chemical, (b) mechanical, and (c) electrical. According to the chemical theory, certain visual substances (or possibly substance such as the visual purple) in the retina are affected by light, and vision originates from metabolic changes produced in these visual substances. The metabolic changes consist of two phases; the upward, constructive or anabolic phase, and the downward, destructive or katabolic change. The visual purple has, in general, played a most important part in chemical theories. But we know that the rate of decomposition and of rebuilding of visual purple is very slow as compared with the act of vision. Edridge-Green states his belief as to the nature of retinal stimulation, as follows: "A ray of light impinging upon the retina liberates the visual purple from the rods and a photograph is formed. The rods are concerned only with the convergence of light impulses to the brain. The ends of the cones are stimulated through the photo-chemical decomposition of the visual purple by light, and a visual impulse is set up which is conveyed to the brain through the optic nerve fibres. The character of the stimulus and impulse differs according to the wave length of the light causing it. In the impulse itself we have the physiological basis of light and in the quality of the impulse the physiological basis of the sensation of colour."

And again, Edridge-Green assumes that the cones of the retina are insensitive to light but sensitive to the changes in the visual purple. Light falling upon the retina liberates the visual purple from the rods and it is diffused into the fovea and other parts of the rod and cone layer of the retina. The decomposition of the visual purple by light *chemically* stimulates the end of the cones—probably through the electricity which is produced—and a visual impulse is set up.

The mechanical theories are built in large measure upon the basis of resonance phenomena in conjunction with chemical action. It is readily conceivable that a ray of light can cause a chemical decomposition of a substance in which the rhythmic excursions of an atom or atoms from, or round about, the centre of attraction in a molecule are in tune with the waves of light falling on such atoms. Likewise, photographic and mechanical examples afford us illustrations of responses produced in which the periods of stimulating body and responding body approximate each other. For example, we have the sensitivity curve of silver chloride, showing a rise in sensitiveness followed by a decline as the portion of the spectrum used for stimulation is changed.

Houstoun has lately attempted another explanation of colour on the basis of the visibility curve of the eye. He believes that the visibility curve is explicable on the supposition that there exists in the eye a very large number of vibrators with a free period in the green and that these execute forced vibrations under the influence of the light waves and that the amplitude of the forced vibration is a maximum when the free period of the vibrations coincides with the period of the incident light. When the energy of the vibration reaches a critical value, the force attracting the vibrator to its centre snaps; the latter then ceases to absorb light energy and a chemical change takes place. This critical period is not the same for all the vibrators but varies from vibrator to vibrator.

Houstoun's theory, either implicitly or explicitly, contained within itself sufficient to cause me in 1917 to write: "We have here, it seems to the writer, the fundamental principle of the so-called quantum theory applied to explain visual phenomena. According to this conception, energy, E , is radiated in discrete or definite units and is connected with the frequency, ν , and the universal constant h , through the equation $E = nh\nu$. The detachment of one, two, three and so forth (n) electrons will give rise to varying quantities of radiation, which are multiples of a definite energy unit." Prof. Joly of Trinity College, Dublin, has this year published a couple of delightful papers containing a discussion on a quantum theory of vision.

It may properly be objected at this point that we have jumped over the traces and that we are not discussing a mechanical theory of resonance, but rather that which we are about to discuss in a few moments, namely, an *electrical* theory. But we feel inclined to believe that none of the modern resonance theories, in which vibrators are concerned and in which free and forced vibrations are involved, assumes or implies the assumption of the vibrators as being the rods or the cones *per se*. Nothing, at least, can probably be further from the truth, because a simple calculation shows that if any microscopically observable structure is to resonate in tune with even the longest light wave, it would need to possess a modulus of elasticity several million times greater than that of steel.

We believe that the ultimately successful doctrine as to the mechanism of vision must involve the conception of electricity, for light is an electro-magnetic phenomenon and can react only with electrical or magnetic systems. The essential feature of the photo-ionic (or photo-electric) visual process consists of the assumption that the rods and cones contain certain electrolytes or ionisable substances which, in the absence of light, are ionised to an extent determined by the equilibrium constant of the reversible dissociation action. The introduction of light into the substance alters the value of this constant by increasing the rate of dissociating substance. Such an action would be the result of the absorption of light energy producing an increased ionisation as a volume effect in the body of the receptor cell. Or, possibly, the action is a truly photo-electric one, in that it is a surface effect with the ejection of electrons from the surface of the retina. In the older physics we should write of resonances as a correlation between radiant energy and the chemically dissociated atom. In the light of modern physics we should write and speak of resonance of radiant energy and electrons. In fact, the essential points in the mechanical theory of retinal stimulation as consisting of resonance effects coupled with chemical action fit in with many of the physical phenomena known as photo-electric actions. We are, therefore, presumably dealing, under the tenets presented to us in this theory, with the expulsion of electrons due to resonance; the electrons are set in resonant vibration by the incident light and acquire sufficient velocity to enable them to escape from the atom. From the view point of modern science we may regard it as fairly certain that the first stage of any photo-chemical reaction consists in the separation of negative electrons under the influence of light. It is obvious that the absorption of light is necessary before any change can be brought about.

According to the theory of Grotthaus we may say that the "action of a ray of light is analogous to that of a voltaic cell."

Hence, the electrical theory supposes that the visual impulse is the concomitant of an electrical impulse; that an electrical current is generated in the retina under the influence of light and that this is transmitted to the brain. It is an undoubted fact that light gives rise to electrical retinal current and that, on the other hand, an electrical current properly applied causes the sensation of light. Holmgren, Dewar, McKendrick, Kühne, Steiner, Waller, Gotch, Einthoven and Joly, and Sheard and McPeck, have shown that illumination produces electric variation in a freshly excised eye. Currents of injury must be eliminated and other precautions taken to avoid spurious effects. All investigations are in agreement that the retina, under illumination, becomes more negative than the nerve end, which appears to indicate that the negative ions of the receptor cells diffuse more readily than do the positive ones, and this is just the reverse of the case for the nerve proper. When the retina is stimulated by light, the original potential difference is increased rather than decreased. The action of light is to increase the negativity of the retina. This is exactly what would occur if light increased the ionisation of the substance or substances inside the cells. Or, possibly, the potential difference increase may be due to the ejection of electrons from atoms in the vicinity of the membrane; this action would be strictly analogous to that occurring in the photo-electric effects with metals. Careful experiments by Joly and later by Poole with the black pigment of sheep and oxen and with fresh retinas gave no results indicating a surface emission of electrons. But we seriously question whether any such surface emission should be expected. For, in the eye at least, we have presumably no accelerating potential. Rather do we conceive of the process as one in which volume ionisation takes place and in which diffusivity of negative and positive ions is sufficiently different as to develop a difference of potential between the retina and the first neural station or synapse along the visual path.

Quite recently Sheard and McPeck have carried on investigations as to the retinal current responses to light of varying wave length. The eye in each case was stimulated continuously with a given narrow spectral region for a period of several minutes and the relationships between time of stimulation and increase or decrease of potential difference between the stimulated retina and non-stimulated nerve stump were obtained. Constant stimulation produced fluctuations in the nerve current of about 12 seconds period combined with a progressive change in the potential of the nerve stump with respect to the cornea, this change consisting of an increase in the positive potential of the nerve for the longer visual wave lengths and a decrease for shorter wave lengths lying roughly at $\lambda 5300$. There is evidence, therefore, of two processes operating in the production of retinal electric potentials and that, in general, the cold colours stimulate the production of negative potentials (from retina to nerve) while the warm colours produce positive potentials. Results were obtained showing that practically no retinal potential changes occur, after the neutral period of excitation by light of a particular spectral character, unless exposures are made in a region very closely approximating the complementary

colour and that, when so exposed, the potential is carried back approximately to the value which it possessed prior to the dual exposure. Hence, when a given potential level has been established under a certain spectral stimulant, the complementary in turn produces the maximum deviation from this level.

In a general way, therefore, the experimentation on the retinal (electric) currents set up in enucleated eyes under light stimulation indicates:

1. That positive retinal currents are manifested and positive electromotive forces established when an eye is illuminated by light.
2. That the source of these electrical changes lies in the retinal structure or in the posterior part of the eyeball.
3. That luminous stimuli of different wave lengths—from red to violet—give initially positive retinal currents. Hence, all responses are of the same general type, and act in accordance with their luminosity.
4. That the stimulus-response relation is such that the curve representing it is concave to the axis representing the stimulus.
5. That fatigue is less pronounced in the case of the retina than in that of muscle.
6. That the time-relations of the responses or periods of latency after application of stimuli are of the order of $1/5$ to $1/2$ a second.
7. That there is a characteristic response to darkness as well as to light stimulation.
8. That the range of light vibrations under which animals' (frogs quite largely) eyeballs give definite photo-electric responses corresponds very closely to the range of vision of our own colour sensations.
9. That photo-electric effects with sensitive cells may be obtained which are wholly analogous to phenomena obtained with eyeballs.
10. That experimental results point to the localisation of two photo-electrical substances in the posterior half of the eyeball or that the retina is the seat of a double electrical movement which may consist of duplex changes in one substance or of two changes in two different components or substances.

With respect to this last conclusion as to the presence of two photo-electric substances or of duplex changes in one substance, we have the very recent work of Selig Hecht on the photo-chemistry of visual purple and human retinal adaptation. The course of retinal dark adaptation, he concludes, is a bi-molecular one; two substances are decreasing in concentration according to the ordinary velocities of chemical reactions. The substance synthesised during dark adaptation is the photo-sensitive substance and the two materials which form this photo-sensitive substance are its precursors as well as its decomposition products. The initial process in visual reception in dim light must, therefore, depend on a reversible photo-chemical reaction in which a photo-sensitive substance is broken down into two decomposition products. During dark adaptation of the retina these two decomposition products then unite to form again the sensitive substance which gave rise to them.

These retinal currents have their counterparts or analogues in non-organic substances when exposed to light. Sensitive inorganic cells may be constructed;

for example, we may take a hollow cup of silver and sensitise the inside with bromine vapour, fill the cup with water and roughly imitate the eye in composition and in having a sensitive receiver and a less sensitive conducting nerve stump. Such a cup, under alternate exposure to light and freedom therefrom, shows a series of response and recovery curves, the counterpart in every wise of the responses and recovery of the retinal currents in frogs' eyes. In general it may be said that the responses of the retina to the stimulus of light are in every way similar to those of certain sensitive inorganic cells. In both we have, under normal conditions, a positive variation; in both the intensity of response, up to a certain limit, increases with the duration of illumination; it is affected in both alike by temperature; in both there is low fatigue effect; the increase of response with intensity of stimulus is small in both; and furthermore, even in deviations from normal responses, such as reversal of response, preliminary negative twitch on commencement, and terminal positive excess on cessation of illumination and decline and reversal under continued action of light, parallel effects are found.

The action of luminous stimulation upon the retina must lie without doubt in the ionisation processes set up in the visual receptor cells. What these processes are, or what the nature of the changes produced is still an enigma. However, ionisation involves a dissociation into positively and negatively charged atoms and electrons. A stimulation by light must, therefore, set up a certain process of ionisation whereby the positive ions may be thought of as moving in general in one direction and the negative atoms (if such exist) and electrons as moving in opposite directions. Photo-electric activities of metals, such as sodium, show a discharge of negative electrons under the action of light, the rate of discharge depending upon the metal and the character of the illumination. However, we need not think of motion of both sets of ions but of one only, for example, the electrons, and that membranes and tissues are more permeable to one than to the other of the ionised products. We may suppose that substances in the absence of light are ionised to a degree dependent upon the equilibrium constant of the reversible dissociation action. The introduction of light into the system alters the value of this constant by increasing the rate of the dissociating change without affecting that of reassociation. It seems not improbable, therefore, that the passage of one set of charged particles out of the receptor cells along nerve trunks will thereby increase the potential difference between retina and nerve and cause a building up of a surplus of the second kind or set of charged products of the ionisation process which, by accumulation, will inhibit, and in large measure sweep out, the oppositely charged products of dissociation when the concentration becomes sufficient.

Using these facts and others Troland has evolved a theory of visual impulse as consisting of the actual propagation of the positive ion from the rod and cone cells along the optic nerve and tract to the cerebellum and that the propagation takes place with the speed of the visual impulse. The propagation occurs within neuro-fibres which are thought of as *molecular tubes* within which it is possible for ions to travel without encountering great resistance. And Troland leads us through

to his conclusion as to the general nature of visual cerebrosis from this account of the mechanism of the impulse and postulates that "the cerebral state corresponding with any condition of retinal stimulation consists simply in the presence in the cerebral cells of the specific ions (or electrons) which are liberated in the retina by the action of the light."

And H. S. Allen had evidently in mind somewhat the same notions in his communication to the Roentgen Society in 1919. For he believes that it is unnecessary to assume that a photo-chemical change is the cause of the visual sensation; it is, he says, "sufficient to suppose that photo-electric action takes place in the chemical substance or substances contained in the rods or cones, so that we have a separation of electrons resulting in electrification of the nerve cells, which sets up the nervous impulse to the sensorium. If this view is correct, the changes in the retina are identical with those which take place in a phosphorescent substance or with those which take place in the formation of a latent image on the photographic plate." And the fact that the form of the curve which shows the relation between the sensitiveness of the retina to light of different wave lengths bears a very close resemblance to the curve which shows the variation of photo-electric activity with wave length, is also in support of this contention. But, on the other hand, Coblenz, of the Bureau of Standards, concludes that, in his opinion, evidence is not sufficient to warrant the belief that there is a connection between the phenomena of colour perception and brightness and the phenomena of photo-electric sensitivity of inanimate material, and that, considered as a whole, spectro-photo-electric sensitivity in solids is only vaguely, if at all, similar to visual response.

We may, therefore, in conclusion—being of the firm belief that the visual receptor process is of a character included within the broad term photo-electric—review some points with reference to the (quantum) theory of vision as presented recently by Prof. Joly. The fundamental assumption is, very naturally, that the origin of brightness and colour vision is to be sought in the liberation of electrons from a photo-sensitive substance or substances. The rhodopsin is such a substance. Furthermore, the spectral range of vision is in excellent agreement with the spectral absorption curve of the visual purple, and this is to my mind one of the most enticing evidences that rhodopsin plays a very important part in some manner in vision. It may simply form the intermediary or catalyser between light and the nerve.

And again, the rods contain the sensitiser in the form of rhodopsin. The electrons set free by light action expend their kinetic energy in stimulating the nerve and doubtless establish an electronic current into the ganglion cell or synapse with which the nerve makes connection. If the rods very completely absorb light energy, there will be a free and copious development of electrons; a confused flow of stimuli to the nerve—some with low and some with maximum kinetic energy—will follow, too mixed and too crowded to permit of analysis. The sensitivity of rod vision is indeed extraordinary. Henri and de Bancels have shown that the retina is sensitive to an amount of light energy of the value of 5×10^{-13} erg. The quantum for green light is practically this figure. We may believe, therefore, that one quantum is sufficient to excite vision: that is saying that the liberation of a single electron by

green or blue light can excite visual sensation. It is easily shown that in viewing a standard candle at the distance of 3000 metres some five to ten electrons are liberated in each rod per second. There is of course absorption of energy which may be expended in other ways than the ejection of electrons. But it is evident that a few electrons (at least one) must be conserved so as to make their way to the optic nerve. Hence, the stimulus arising from one quantum must constitute an appreciable portion of the threshold stimulus.

The cones are structurally different in that they contain no visible quantity of visual purple. Doubtless the outer segment of the cone is bathed in a photo-sensitive fluid—probably rhodopsin. Here the light is absorbed, either at the surface of the cone or within the thin layer separating cone from cone. Some electrons produced by light stimulation do not escape from the sensitiser. The fastest, however, carry into the nerve the full quantum of energy which is characteristic of the frequency giving rise to them. If the intensity of the light is considerable there would be many of these electrons which would be the most effective in exciting a nerve stimulus. Colour vision is in abeyance at very low luminosities; this may be explained on the basis that there are insufficient of what we may call "characteristic" electrons to excite colour sensation. And again, when colour is excited, there will always be a dilution of white because of the presence of the slower-moving electrons and "non-characteristic" of the frequency which gives rise to them. Hence quantitative sensitivity may be assigned to the rods and qualitative sensitivity to the cones. In one case the sensitiser is located within the nerve and in the other is located outside of the nerve. "The sensory stimulus emanating from the rods is compounded of many sources of stimuli, i.e. such as may originate in electrons possessing every velocity and kinetic energy up to the maximum proper to the frequency of the activating light. The stimulus emanating from the cones, on the other hand, is purified of all stimuli save those arising from the kinetic energy of electrons which are activated by the energy of absorbed quanta. The electrons are, in fact, selectively presented to the nerve; all other sources of stimuli take place outside the cone and are cut off from it by the filament which invests it. In the cone the more intense stimuli tap out to the brain the sensation of colour which we associate with the intensity of the quantum involved. To this succession of characteristic nerve impulses there is added an underlying accompaniment; the white or luminous sensation made up of all the feebler electrons which impart to the nerve but a fraction of that which is characteristic of the frequency of the light entering from the world without." (Joly, *Phil. Mag.* Feb. 1921.)

And as we draw this oration to a close we feel that the enigma of vision is still awaiting solution. Unfortunately the science of physiological optics, in so far as it deals with the correlation of the acts of visual consciousness with material factors and processes, has not reached the stage of development comparable with its sister science, physical optics. For our theories of vision are so frail and so puny as compared with the exactness of the mathematical sciences. Such must, perhaps, always be the lot and fortune of that which deals with the animate in contradistinction to the inanimate. All the more necessity, therefore, for bringing to the

solutions of such problems the trainings of the physicist, the physiologist, and the psychologist. For it must be quite obvious to us all that the explanation of the phenomena of vision in their entirety must involve a knowledge of the laws of the brain and its operation, the functions and structure of nerve tissues, and the physical chemistry of the retinal processes. We plead for a closer coordination of these sciences and we hope that the future will be the possessor of investigators of wide acquaintance with these fields, so that the theories of the physicist, for instance, may not, as they are propounded to explain phenomena involving animate material—flesh and blood and all that these terms include—be of no avail because of their failure to take into consideration physiological and psychological facts. We may not hope for many—if any—geniuses of the future so fortunate as to embody the full complement of all that we have asked for, since such an enormous expansion of knowledge has taken place since the time of Young, but we can hope for a closer coordination and association in experimental work to the end that kindred minds may work together, each presenting a different view point and each serving as a stimulus and a check at the same time.

Thomas Young was one of the intellectual giants of the scientific world, for his work shows us his wonderful insight into physical laws and phenomena, his wide acquaintance with the principles of medicine and physical diagnosis as then developed is portrayed in his *Practical and Historical Treatise on Consumptive Diseases*, and his knowledge of the workings of the human mind is portrayed in his writings on insanity, inaccurate as we now know them to be. These are but evidences of a catholicity which is unique. We of average ability are constrained to concentrate our attention to very narrow regions of investigation, but this limitation enables us all the more to admire and look with wonder upon that master mind, able, with power and profit to his own and succeeding generations, to range over the whole wide field of human knowledge. So may we continue to reverence the memory of Thomas Young by being "doers as well as hearers of the word."

THE PATH OF RAYS IN PERISCOPES HAVING AN INVERTING SYSTEM COMPRISING TWO SEPARATED LENSES

BY DR ALEXANDER GLEICHEN

(Communicated by Dr James W. French)

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SYNOPSIS. Foreword. Principal elements. The marginal beam, field of view, and magnification. Ray tracing as applied to periscopes. The principal rays and the centre of the effective stop. Illumination of the marginal beam and its relation to the field of view. The Collector Law. Determination of the collector lens. Periscopes with parallel path of principal rays in front of the eyepiece. Special numerical example. Calculation of the Ramsden eyepiece. Reduced top periscopes. The external form of the submarine periscope.

(1) FOREWORD

PERISCOPES constitute a special class of instrument which finds important use in the services of the Army and the Navy, in the latter case more especially as periscopes for submarines. The characteristics of these instruments are their comparatively great length and small diameter. For these reasons there are imposed on the space occupied by the rays certain restrictions which must be overcome by a proper arrangement of the optical elements. The periscope is a special form of the telescope. An objective forms an inverted image of the distant object in its focal plane. This image is then erected by an inverting system and at the same time is transferred through the whole length of the tube to the eyepiece end where it is to be observed through an astronomical eyepiece (or, in the simplest case, a single convergent lens). The inverting system consists of two convergent lenses between which the rays of light are parallel, that is, the image formed by the objective is situated in the focal plane of the first inverting lens.

In the following discussion it is intended to show how the magnification, the field of view, and the illumination in the centre and at the edge of the field of view depend on the length and diameter of the instrument, in order to derive therefrom the most advantageous optical arrangement.

At the request of Messrs C. P. Goerz in Berlin-Friedenau the author has carried out a theoretical investigation of the path of the rays in periscopes of the type indicated and has the permission of the firm to publish it. The results obtained provide in a simple manner a solution of all the optical questions concerning the path of the rays in submarine periscopes and permit of periscopes being constructed to fulfil specified conditions.

For this purpose it is sufficient to assume that the lenses of the instrument are infinitely thin in the known sense of geometrical optics. In the accompanying figures these lenses in several cases are therefore represented simply by straight lines

perpendicular to the axis whose lengths give at the same time the apertures of the lenses. In practice these lenses are replaced (except in the case of the so-called collector lenses) by cemented achromatic combinations and consequently the correction of the well-known aberrations is carried out principally by the usual methods of computation for telescopic lenses. If the periscope is intended for submarine use or to see over obstructions, the tube is placed vertical with reflectors inclined at 45° to the horizontal plane at the objective and eyepiece, these reflectors having generally the form of total reflecting prisms. Consequently the observer looks into the instrument in a horizontal direction. Such an arrangement is indicated in Fig. 10. In the following discussion these reflectors in the first instance are not taken into account, since they have no influence on the actual path of the rays in the periscope beyond bending them twice through 90° .

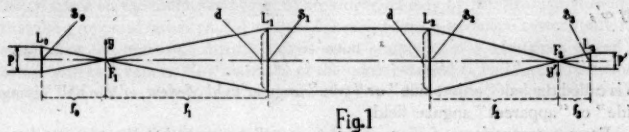


Fig. 1

In Fig. 1 the two inverting lenses are indicated by L_1 and L_2 , the objective by L_0 , and the eyepiece by L_3 . Let the vertices of the lenses (which are assumed to be infinitely thin) be represented by S_0 to S_3 ; the focal lengths by f_0 to f_3 ; the separation between the two inverting systems by $l = S_1S_2$; the overall length by $L = S_0S_3$; and the magnification by v . The objective forms in its focal plane (F_1 plane) an image $2y$ perpendicular to the axis which is transferred by means of the inverting system (L_1L_2) to the focal plane of the eyepiece (F_2 plane) where its magnitude is $2y'$. The quantities p and p' are called the entrance and exit pupils respectively. The illumination depends on the value of p' and is according to the usual convention equal to p'^2 , where p' is expressed in millimetres.

(2) MARGINAL BEAM, FIELD OF VIEW, AND MAGNIFICATION.

In addition to the central beam shown in Fig. 1, the periscope is traversed by oblique beams proceeding from object points lying off the axis. The most oblique beam which passes through the instrument defines the field of view and will be called the "marginal beam." It is determined in the following way.

From the marginal point A of the objective image y (Fig. 2) there are incident

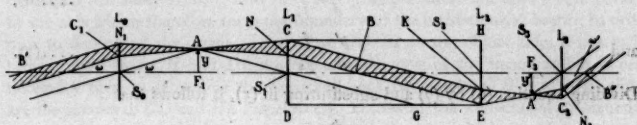


Fig. 2

upon L_1 many rays which after refraction must be parallel to the direction AS_1G . The lengths CD and HE represent the apertures of the two inverting lenses L_1 and L_2 respectively. Let $CD = HE = d$. If now straight lines CK and EN are drawn through C and E parallel to S_1G , there is obtained the portion of the beam $CNEK$ (in Fig. 2 shown shaded) which just passes through the inverting system. If from C and N rays are drawn outwards through A meeting the objective L_0 in C_1 and N_1 and thence are drawn parallel to AS_0 , there is obtained the marginal beam which cuts the optical axis at an angle ω . Since now AS_0F_1 is also equal to ω and $S_0F_1 = f_0$, it follows that

$$\tan \omega = \frac{y}{f_0} \quad \dots (1).$$

Similarly at the eyepiece end, if the inclination of the emergent beam is represented by ω' ,

$$\tan \omega' = \frac{y'}{f_3} \quad \dots (2).$$

ω is called the half "object side" or "true" angular field of view, ω' the half "image side" or "apparent" angular field.

From a comparison of Figs. 1 and 2, it will be seen that the extreme beam emerging from the eyepiece has a smaller cross section than the central beam and therefore smaller illumination.

In so far as $\tan \omega'$ is greater than $\tan \omega$, the object viewed through the periscope will appear magnified; therefore

$$v = \frac{\tan \omega'}{\tan \omega} \quad \dots (3).$$

From equations (1), (2) and (3) it follows that

$$v = \frac{y'}{y} \cdot \frac{f_0}{f_3} \quad \dots (4),$$

and therefore, since

$$\frac{y'}{y} = \frac{f_2}{f_1},$$

$$v = \frac{f_0}{f_1} \cdot \frac{f_2}{f_3} \quad \dots (5).$$

Further, it follows directly from the similar triangles of Fig. 1 that

$$\frac{p}{d} = \frac{f_0}{f_1} \quad \dots (6),$$

and

$$\frac{p'}{d} = \frac{f_3}{f_2} \quad \dots (7).$$

Dividing equation (6) by (7) and substituting in (5), it follows that

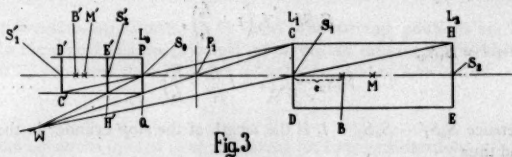
$$\frac{p}{p'} = v \quad \dots (8).$$

Equations (1) to (8) have been deduced for infinitely thin lenses only. They are generally true, however, for any lenses. In particular equations (1) and (2) are true even if the objective or the eyepiece comprises several separated lenses.

(3) RAY TRACING.

The theory of ray tracing as generally given in textbooks is not directly applicable to periscopes such as are under consideration, because neither the entrance pupil nor the exit pupil is the basis of all the beams passing through the system and there is really no effective stop in the sense usually understood in the theory of ray tracing. Yet it is practicable even for the case in question to define the idea of the principal ray and the centre of the effective stop, as will be shown in the following discussion.

Since the space between L_1 and L_2 on the one hand and the object space and image space on the other hand (Fig. 2) are traversed only by parallel rays, it follows that the groups of lenses (L_0L_1) and (L_2L_3) each forms a telescope system with the result that the periscope can be looked upon simply as two telescopes placed in series with each other. Now an image of the space between L_1 and L_2 can be formed on the left and on the right through the objective and eyepiece respectively and a



"stop cylinder" will be obtained as a real image of the space bounded by $CHDE$ (Fig. 2), in the object space and in the image space. All the parallel beams which pass from L_1 to L_2 will also proceed within the boundaries of these stop cylinders of parallel light, although with correspondingly altered obliquities. It will be evident also that the two stop cylinders bear to each other the relation of object and image. The diameters of these cylinders are equal to those of the entrance and exit pupils respectively of the periscope. The lengths of the cylinders can easily be found either by diagrammatic construction or by calculation.

As an example, the construction for the stop cylinder on the object side is illustrated in Fig. 3. In the first instance rays from C and D are drawn as in Fig. 1 through F_1 to meet L_0 in Q and P . From Q and P there are drawn rays parallel to the axis which therefore form the boundary of the incident axial beam. In order now to determine the image of the space $CDHE$ on the object side, it has to be remembered that CH and DE are the extreme rays of the image and it is only necessary then to determine the front and back boundaries $C'D'$ and $H'E'$ which are the images of CD and HE respectively. Following the well-known construction for thin lenses there is drawn through C and S_0 a straight line which intersects at C' the lower ray drawn parallel to the axis. Perpendicularly over this point lies

the point D' . In order to find H' there is drawn through H and S_1 a straight line which cuts the line CF_1Q produced in W and the line WS_0 cuts $C'Q$ in H' . As before, the point E' lies perpendicularly over H' . In a similar way the stop cylinder in the image space may be constructed. It should be observed that in Fig. 3 the points S_1' and S_2' are the images of the lens vertices S_1 and S_2 . If the middle point of S_1S_2 is called M , then the image M' of M will also be the middle point of the length $S_1'S_2'$.

The stop cylinders can also be readily determined by calculation. The diameter of the cylinder on the object side (Fig. 3), that is, the length $PQ = E'H' = D'C'$, is represented by p . If a point B is taken on the axis between S_1 and S_2 then the image B' of B will lie between S_1' and S_2' . Representing S_1B by c , then

$$S_0B' = f_0 \left(1 - \frac{c \cdot f_0}{f_1^2} + \frac{f_0}{f_1} \right) \quad \dots (9),$$

as can be easily deduced from the simple optical formulae.

From (9) it follows, for example,

for $c = 0$,

that

$$S_0S_1' = f_0 \left(1 + \frac{f_0}{f_1} \right),$$

and for $c = l = S_1S_2$,

that

$$S_0S_2' = f_0 \left(1 - l \frac{f_0}{f_1^2} + \frac{f_0}{f_1} \right) \quad \dots (9a).$$

The difference $S_0S_1' - S_0S_2' = l_1$ is the length of the stop cylinder in the object space, and thus

$$l_1 = l \left(\frac{f_0}{f_1} \right)^2 \quad \dots (10).$$

The equation (9) serves also for the calculation of the stop cylinder in the image space (on the eyepiece side) when the quantities f_0, f_1 , and c are replaced by f_2, f_3 , and $l - c$ respectively. If the image points are denoted by double dashes (for example B'' as the image of B), then

$$S_2B'' = f_2 \left(1 - (l - c) \frac{f_2}{f_3^2} + \frac{f_2}{f_3} \right) \quad \dots (10a).$$

For $c = l$,

$$S_2S_4'' = f_2 \left(1 + \frac{f_2}{f_3} \right),$$

and for $c = 0$,

$$S_2S_1'' = f_2 \left(1 - l \frac{f_2}{f_3^2} + \frac{f_2}{f_3} \right),$$

$$S_2S_4'' - S_2S_1'' = l \left(\frac{f_2}{f_3} \right)^2.$$

From equation (5) also

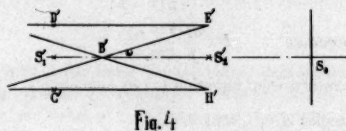
$$\frac{l_2}{l_1} = v^2.$$

(4) THE PRINCIPAL RAYS AND THE CENTRE OF THE EFFECTIVE STOP.

Illumination of the Marginal Beam.

In Fig. 4 the stop cylinder in the object space is again shown. If any chosen point B' on the axis is taken to represent the centre of the entrance pupil, then all rays passing through this point are called principal rays. These principal rays pass also through the point B which lies on the axis between S_1 and S_2 (Fig. 3). The point B is called the centre of the effective stop.

If a principal ray is imagined to be a thin rod capable of rotation about the point B' (Fig. 4), then this rod can take up all possible positions within the stop cylinder,



whose dimensions are assumed to be fixed. The extreme positions are $B'E'$ and $B'H'$. In these positions the principal ray forms the extreme ray, so that $E'B'S_1'$ and $H'B'S_2'$ represent the half object side angular field of view ω and

$$\tan \omega = \frac{E'S_1'}{B'S_1'}.$$

Now from equations (9) and (9 a) and taking (6) into consideration,

$$B'S_1' = S_0B' - S_0S_1' = (l-c) \frac{f_0^2}{f_1^2} = (l-c) \frac{p^2}{d^2}.$$

Since, further, $E'S_1' = \frac{p}{2}$, it follows that

$$\tan \omega = \frac{d^2}{2p(l-c)} \quad \dots (11),$$

a relationship which forms the foundation for the further discussion of the subject.

The marginal beam which passes through the point E' (Fig. 4) proceeds in the space between L_1 and L_2 through the conjugate point E . In Fig. 2 it is represented by the ray $B'N_1ANEAN_2B'$. In this figure $S_1B = c$.

Let us now in Fig. 2 make

$$\frac{CN}{2 \cdot CS_1} = \mu \quad \dots (11 a).$$

From similar triangles in Fig. 2 it follows that

$$\frac{S_1N}{S_1B} = \frac{S_2E}{S_2B} \quad \text{or} \quad \frac{S_1N}{c} = \frac{d}{2(l-c)},$$

and

$$S_1N = \frac{d \cdot c}{2(l-c)}.$$

Now $CN = CS_1 - S_1N = \frac{d}{2} - \frac{d \cdot c}{2(l-c)}$.

From equation (11 a) it accordingly follows, since $CS_1 = \frac{d}{2}$, that

$$\mu = \frac{1}{2} \left(1 - \frac{c}{l-c} \right) \quad \dots (11 b).$$

If, further, we make

$$\frac{c}{l} = m \quad \dots (12),$$

there is obtained from equation (11 b) after several transformations

$$1 - m = \frac{1}{2(1-\mu)} \quad \dots (12 a),$$

and equation (11) becomes

$$l = \frac{d^2(1-\mu)}{p \cdot \tan \omega} \quad \dots (13).$$

The value μ is determined from equation (11 a), namely $\mu = \frac{CN}{d}$. But in Fig. 2, from the similar triangles AN_1C_1 and ANC ,

$$\frac{C_1N_1}{CN} = \frac{f_0}{f_1} = \frac{p}{d}.$$

Therefore

$$\frac{C_1N_1}{p} = \frac{CN}{d} = \mu \quad \dots (13 a).$$

Similarly it follows, since in Fig. 2, $EK = CN$, that

$$\frac{C_2N_2}{p'} = \mu \quad \dots (13 b).$$

By the above calculations there has been determined a centre of the effective stop, and the points B' and B'' which serve as the centres of the entrance and exit pupils respectively are the points of intersection of the principal rays in the object and image spaces. The position of the first is important, especially for the conditions of the periscope to be discussed later, and the position of the latter determines the "eye-point." The departure from the usual teaching regarding ray tracing consists in this, that the position of the point B is not *a priori* determined by the system but can be chosen anywhere within certain limits. The choice is based on the following considerations.

The principal rays, as defined above, are no longer the centre of gravity rays of the beams of light. They only coincide with the latter in the case of the beam incident parallel to the axis. The greater the obliquity at which a beam passes through the system, the more does the principal ray depart from the symmetrical position. In the marginal beam, in fact, it forms the outer boundary of the beam (see for example Fig. 2).

The magnitude μ defined above from equations (13 a) and (13 b) is simply the ratio of the diameter of the marginal beam to the diameter of the axial beam, both on the object side and on the image side of the periscope. The magnitudes C_1N_1

and C_2N_2 represent, as it were, the diameters of the entrance pupil and of the exit pupil for the oblique beam. (The cross section of this oblique beam is not circular in form, as in the case of the axial beam, but has the form of the segments of two intersecting circles.)

The magnitude μ is therefore a measure of the illumination of the side of the field relatively to that of the centre. If, for example, $\mu = \frac{1}{2}$ when $p' = 6$ mm., then it follows that the emerging oblique beam has a cross-sectional dimension of only 2 mm. To what value of μ it is permissible to descend without making the loss of light at the side of the field uncomfortably noticeable to the observer is a matter of experience. If, for example, the magnitude C_2N_2 is chosen equal to 1 mm. then, when the same unit of measurement is used throughout, it follows from (13 b) that

$$\mu = \frac{1}{p'} \quad \dots (13 c).$$

If a certain value of μ has been decided upon, then equation (13) provides a definite relationship between the quantities l , d , p and $\tan \omega$. Further, equations (12) and (12 a) give the value of c and therefore the centre of the effective stop expressed in terms of μ and l . By means of equations (9) and (10 a) the positions of the entrance and exit pupils can also be found.

(5) ILLUMINATION OF THE MARGINAL BEAM IN RELATION TO THE FIELD OF VIEW.

Referring to Fig. 4, it will be seen that, if the point B' is displaced from S_1' towards the right, the angle ω will increase until B' coincides with the centre-point M' (Fig. 3) of $S_1'S_2'$. From this point the angle ω again decreases and at S_2' it has the same value as in S_1' over M towards S_2 . If the angle ω increases, the illumination of the marginal beam is reduced, as is evident from equation (13). If B' coincides with S_1' , that is, B with S_1 , then $c = 0$. From equation (11 b) it follows then that $\mu = \frac{1}{2}$. On the other hand, if B' coincides with M' and therefore B with M , then $c = \frac{1}{2}$, and from equation (11 b) $\mu = 0$. The illumination of the extreme beam is therefore zero, while the angle ω has reached its greatest value.

(6) THE COLLECTOR LAW.

Up to this point it has been assumed that the beam of rays through the space between L_1 and L_2 of diameter d is subject to no limitation and that more particularly in the F_1 and F_2 planes there are no diaphragms that could restrict the path of the rays. These assumptions are justified because by placing a lens in the F_1 plane or in the F_2 plane, it is possible to control the path of the rays between L_0 and L_1 and between L_2 and L_3 in any desired way. These intermediate lenses are called collector lenses. They will be denoted by L_v and L_c and their focal lengths by f_v and f_c , the former being placed at F_1 and the latter at F_2 . In order to explain the effect of these lenses it is necessary to use a law of geometrical optics which may be called the "collector law." This law can easily be derived from the fundamental principles of optical image formation and may be expressed as follows.

The positive lens L of focal length f_1 in Fig. 5 forms a real image z' at A' of the object z . If now a second lens of focal length f_2 is placed at F , the first focal plane of L , then as a result the image z' will be moved from A' to A'' and consequently

(1) The size of the image z' remains unaltered.

(2) The shift of the image has the value

$$A'A'' = \frac{f_1^2}{f_2}.$$



It is especially worthy of note that the displacement $A'A''$ is independent of the position of the object z . The displacement is, therefore, the same for all objects at any distance. From Figs. 1 to 3, therefore, it follows that, by the introduction of a collector lens of focal length f_v in the F_1 plane, the stop cylinder on the object side, together with the centre B' of the entrance pupil, will be moved towards the objective by the length $\frac{f_0^2}{f_v}$, while the cross section of the cylinder and therefore also the diameter of the entrance pupil remains unaltered. Similar results hold for the eyepiece side when a collector lens of focal length f_e is introduced at the F_2 plane. The collector lens, together with the eye lens, forms then an eyepiece of the Ramsden type. The size of the exit pupil remains unaltered and the centre of the exit pupil is moved towards the eye lens by the amount $\frac{f_3^2}{f_e}$.

Further, the angles ω and ω' remain unaltered by the introduction of the collector lens and also the size of the images y and y' ; and thus the equations (1) and (2) remain unaltered and in general all the relationships developed above.

The two collector lenses introduced in the F_1 and F_2 planes will be called L_v and L_e , their focal lengths f_v and f_e , and their apertures ψ_v and ψ_e respectively.

(7) THE DETERMINATION OF THE COLLECTOR LENS.

In Fig. 6, S_0 is the vertex of the objective plane and B' is the centre of the entrance pupil when there is no collector lens. From equation (9),

$$S_0B' = f_0 \left(1 - \frac{c \cdot f_0}{f_1^2} + \frac{f_0}{f_1} \right).$$



The effect of the collector lens will be to move the centre of the entrance pupil to R and therefore

$$B'R = \frac{f_0^2}{f_y}$$

It follows that, if $RS_0 = \lambda$,

$$\lambda = f_0 \left(1 - \frac{c \cdot f_0}{f_1^2} + \frac{f_0}{f_1} \right) - \frac{f_0^2}{f_y} \quad \dots (14).$$

Since now the marginal ray passes through R with the inclination ω ,

$$\tan \omega = \frac{\psi_0}{2\lambda} \quad \dots (15),$$

where ψ_0 denotes the whole aperture of the objective.

In general ψ_0 must be smaller than the available diameter of the tube and greater than (or equal to) p , since otherwise the central illumination would be reduced. When the greatest possible use is made of the available space,

$$\psi_0 = p \quad \dots (15 a)$$

and therefore

$$\tan \omega = \frac{p}{2\lambda} \quad \dots (16),$$

an equation which serves for the determination of λ .

Half the aperture ψ , of the collector lens must be at least as great as the image y . In the limit, therefore, from equation (1),

$$\psi_y = 2f_0 \tan \omega \quad \dots (17).$$

Similar considerations apply to the eyepiece side.

There is no purpose in considering further in a general way the various requirements which determine the construction of a periscope. The special use of the formulae will be exemplified by important particular cases.

(8) PERISCOPES WITH PARALLEL PATH OF THE PRINCIPAL RAYS IN FRONT OF THE EYEPiece.

In Fig. 2, B is the centre of the effective stop (the crossing point of the principal rays) and $S_1B = c$ and $S_1S_2 = l$.

In order to utilise the space the case in which the principal rays in the space between L_2 and L_3 are parallel to the axis is specially important, because the periscope is seldom reduced at the eyepiece end. In this case B must be the focal point of the lens L_2 and $BS_2 = l - c = f_2$. From equations (12) and (12 a) it follows that

$$f_2 = \frac{l}{2(1-\mu)} \quad \dots (18),$$

a relationship which should be added to those already deduced.

In the present case $y' = d$, which is obvious without explanation. The centre B' of the exit pupil coincides with the focal point of the lens L_3 .

If it is assumed that the values $\tan \omega$, p' , v , and d are given and that accordingly $\tan \omega'$ and p are also known, then the following equations serve to determine the focal lengths and separations of the lenses:

$$l = \frac{d^2(1-\mu)}{p \cdot \tan \omega} \quad \dots (I),$$

$$f_2 = \frac{y}{2(1-\mu)} \quad \dots (II),$$

$$f_3 = \frac{f_2 p'}{d} \quad \dots (III),$$

$$\frac{f_0}{f_1} = \frac{p}{d} \quad \dots (IV).$$

If also the objective has the form of a thin lens, then

$$L = f_0 + f_1 + f_2 + f_3 + l \quad \dots (V).$$

From these equations there is easily obtained the equation

$$f_3 = \frac{d}{2 \tan \omega'} \quad \text{and} \quad y' = \frac{d}{2} \quad \dots (Va).$$

If it is also assumed that L and μ are given, there are then five equations involving the five unknowns, f_0 , f_1 , f_2 , f_3 , and l . It must be remembered, however, that y must not be greater than $\frac{d}{2}$ since otherwise the periscope could no longer have the form of a smooth tube but would require to be increased in diameter at the objective end.

From the above equations it follows also that

$$v = \frac{p}{p'} = \frac{f_0 \cdot f_2}{f_1 \cdot f_3} \quad \dots (Vb).$$

The calculation of y in the latter case follows from the equation $y = f_0 \cdot \tan \omega$.

Since $\frac{y}{y'} = \frac{f_1}{f_3}$ and $y' = \frac{d}{2}$, it is obvious that f_1 must not be greater than f_3 if the objective image of size $2y$ is to be contained in the second tube. The calculation of the unknown quantities involves equations only of the first order.

The determination of the collector lens follows from the equations given above.

(9) A SPECIAL CASE.

$$(\mu = \frac{1}{2} \text{ and } f_1 = f_2.)$$

For simple field periscopes, for example for use with mine-throwers, the military authorities specify the length L , the field of view ($2 \tan \omega$), and the exit pupil p' , while no definite requirements are specified regarding the diameter d . In order to obtain suitable lens apertures, it may be conveniently chosen that $\mu = \frac{1}{2}$ and $f_1 = f_2$.

As the equations of the preceding section show, S_1 is then the crossing point

of the principal rays and $f_1 = f_2 = l$. By the elimination of the focal length and the quantity l , the equations of the preceding section give

$$d^2 + \frac{d(v+1)p'}{3} - \frac{2 \tan \omega' \cdot L \cdot p'}{3} = 0 \quad \dots (19).$$

From this quadratic equation d may be found. The other values in question are given by the equations

$$f_3 = \frac{d}{2 \tan \omega'} \quad \dots (19 a),$$

$$f_0 = v \cdot f_3 \quad \dots (19 b),$$

$$f_1 = f_2 = \frac{f_3 \cdot d}{p'} \quad \dots (19 c),$$

which follow from the equations of the preceding paragraph.

For example, let it be given that

$$v = 4,$$

$$p' = 4 \text{ mm.},$$

$$L = 1000 \text{ mm.},$$

$$\tan \omega' = 0.36, \text{ therefore } \tan \omega = \frac{\tan \omega'}{4} = 0.09 = \frac{9}{100}.$$

If these values are substituted in equation (19) and the quadratic equation is solved, then it will be found that

$$d = 27.83,$$

and from equations (19 a) to (19 c) it follows that

$$f_0 = 154.60,$$

$$l = f_1 = f_2 = 268.9,$$

$$f_3 = 38.65.$$

The size of the image $2y$ formed by the objective is in the above case equal to d and may be obtained from $\frac{y}{y'} = \frac{f_1}{f_3}$.

(10) CALCULATION OF THE RAMSDEN EYEPIECE FOR THE CASE IN WHICH THE PRINCIPAL RAYS IN FRONT OF THE EYEPIECE PROCEED PARALLEL TO THE AXIS.

It is a condition now that the principal rays in front of the eyepiece proceed parallel to the axis. Instead of using the equations of section (7), it is preferable on account of the simplicity and obvious nature of the above case to proceed directly. In Fig. 7 the eye lens is indicated by L_2 with its vertex at S_2 . The extreme rays AD and EG parallel to the axis unite in the focal point F' of the lens L_2 . The collector lens L_1 with focal length f_1 is now introduced at F_1 , and in consequence the point F' will be shifted towards R' so that from the collector lens law

$$R'F' = \frac{f_1^2}{f_2}.$$

If the eye-point distance $S_2R' = \lambda'$, then

$$\lambda' = f_2 - \frac{f_1^2}{f_2} \quad \dots (20).$$

Since further the rays DF' and D_1R' are parallel to each other and GF' and G_1R' are also parallel and cut the axis at the angle ω' , therefore, if the aperture D_1G_1 of the eye lens is represented by ψ_3 ,

$$\tan \omega' = \frac{\psi_3}{2\lambda'} \quad \dots(21).$$

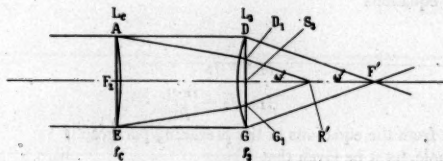


Fig.7

If now the eye distance λ' is definitely known, then from equations (20) and (21) the values ψ_3 and f_c may be determined. In the example of the preceding section, if $\lambda' = 20$ mm., then it follows, since $\tan \omega' = 0.36$, that $\psi_3 = 40 \times 0.36 = 14.4$ mm. The value of ψ_3 must be increased on both sides by $p'\mu$, i.e. by the total amount of $2p'\mu$, in order that the marginal beam may pass through completely. In the above case $\mu = \frac{1}{2}$ and $p' = 4$; therefore ψ_3 has to be increased by 4 mm., i.e. it becomes 18.4 mm. Further, equation (20) gives

$$20 = 38.65 - \frac{38.65}{f_c},$$

from which $f_c = 80$, approximately. The aperture of the collector lens L_c must obviously be equal to 27.83 mm.

* Note. The collector lens L_c on the objective side (at F_1) can be determined from the information in section (7).

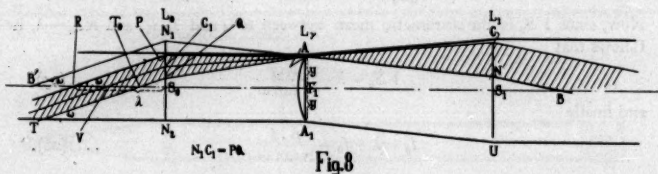
(11) REDUCED DIAMETER PERISCOPES.

If it is necessary to reduce the periscope diameter at the objective end, this can only be accomplished by making the value of the objective image $2y$ smaller than d . Therefore

$$2y = d_0, \text{ where } d_0 < d \quad \dots(22).$$

To the thinner cylindrical part there may be added a conical portion which has at its largest part the diameter d . In the following discussion the simple assumption will be made that d_0 and d represent the constant daylight diameters available in the portions of the periscope. In practice this assumption is not always quite permissible because the internal mechanical parts and variations of the thickness of the wall cause exceptions in periscopes for submarines. Further it will be assumed that the objective has the form of a thin lens (an assumption which is not always true).

Such a periscope is shown in Fig. 8. B is again the centre of the effective stop. The marginal ray emanating from B is refracted by the lens L_1 at N , passes through the edge of the objective image at A , and would, if there were no collector lens placed at F_1 , pass through N_1 to B' . The beam of light to which this marginal ray belongs is CAN . It would meet the lens L_0 as the beam C_1AN_1 and would emerge in the object space as the parallel beam $N_1B'C_1V$ of diameter μ . The heavy black lines indicate the available space. The beam just described passes outside this space



(at N_1) and must be brought within it by means of a collector lens. This collector lens L_1 is placed at F_1 and as the aperture $\psi = 2y = d_0$, the points N_1 and C_1 are then displaced to P and Q such that

$$N_1C_1 = PQ = \mu \cdot p \quad \dots (23).$$

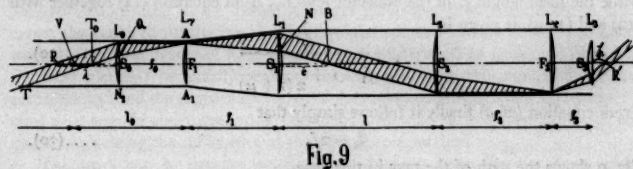
The shaded portion indicates the path of the rays produced by the collector. The marginal ray through P passes through the middle point R of the entrance pupil, and thus

$$RS_0 = \lambda.$$

The thin cylindrical part of the tube could without restricting the path of the rays be produced to the point T . When an entrance reflector or prism, which should be of the smallest possible dimensions, is used (see Fig. 10), it should be placed at the point where the beams in front of the objective cross in the neighbourhood of the entrance pupil. The ray QT may cut the axis in T_0 . In such a case the middle point of the reflecting mirror (hypotenuse of the prism) is made to coincide with the point V which bisects the distance T_0R ; thus

$$l_0 = F_1V = VS_0 + f_0 \quad \dots (24),$$

which gives the length of the reduced cylindrical part. Fig. 9 shows this relation more clearly.



From Fig. 8

$$\tan \omega = \frac{PS_0}{\lambda}.$$

Further, $PQ = N_1C_1 = \mu \cdot p$, and therefore

$$QS_0 = PS_0 - PQ = \lambda \tan \omega - \mu \cdot p.$$

Now

$$\tan \omega = \frac{QS_0}{T_0S_0}; \text{ therefore } T_0S_0 = \lambda - \frac{\mu \cdot p}{\tan \omega}.$$

Now, since VS_0 is the arithmetic mean between RS_0 and T_0S_0 , and $RS_0 = \lambda$, it follows that

$$VS_0 = \lambda - \frac{\mu \cdot p}{2 \tan \omega} \quad \dots (25),$$

and finally

$$l_0 = \lambda + f_0 - \frac{\mu \cdot p}{2 \tan \omega} \quad \dots (26).$$

If, however, the centre of the reflecting mirror is brought to R , which is quite practicable, then it follows simply that

$$l_0 = \lambda + f_0 \quad \dots (26 a).$$

The value of λ is determined from equation (14) together with (11 b), while a value is assumed for f_1 which is sufficiently small to keep the point N_1 (Fig. 8) within the limits of the periscope.

(12) REDUCED PERISCOPE WITH PARALLEL PATH OF THE PRINCIPAL RAYS BETWEEN L_0 AND L_1 .

If in Fig. 8, by the introduction of the collector lens L_1 , the point N_1 is brought within the periscope, the principal rays between L_0 and L_1 converge in general towards the left, and thus λ has a smaller value than f_0 . But since λ should be as large as possible, the best case is that in which the principal rays between L_0 and L_1 are parallel to the axis, in which case R coincides with the focal point of L_0 . Then

$$\lambda = f_0 \text{ and } 2y = d_0 \quad \dots (27),$$

and from equation (26),

$$l_0 = 2f_0 - \frac{p \cdot \mu}{2 \tan \omega} \quad \dots (28),$$

while the focal length f_1 of the collector lens L_1 , from equation (14) together with (12) and (12 a), is given by

$$f_1 = \frac{f_0^2}{f_0 - 1 + \frac{1}{2(1 + \mu)}} \quad \dots (29).$$

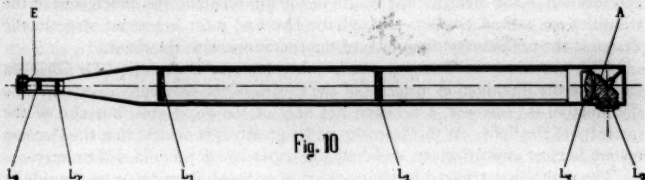
From equation (26 a) finally it follows simply that

$$l_0 = 2f_0 \quad \dots (30).$$

Fig. 9 shows the path of the rays in this case.

(13) THE EXTERNAL FORM OF THE SUBMARINE PERISCOPE.

Fig. 10 shows the external form of the periscope for a submarine. The tube is mounted vertically. L_0 is the objective which naturally must be corrected for spherical and chromatic aberrations in the same way as a telescope objective, while the corresponding collector lens L_7 generally consists of a simple lens. The inverting lenses L_1 and L_2 are also represented as achromatic combinations. The eyepiece consists of an achromatic eye lens L_3 and a simple collector lens L_4 (the so-called Kellner type of eyepiece). The entrance and exit reflectors are two total reflecting isosceles right-angled prisms, E and A .



DISCUSSION

Dr J. W. French: Dr Gleichen needs no introduction. For many years he occupied a prominent position in the German Patent Office, and in that capacity acquired a very comprehensive knowledge of optical science and instruments. In Britain he is most widely known as an author, and one of his most excellent books has been translated into English. At the present time Dr Gleichen is associated with Messrs C. P. Goerz of Berlin who have done so much for the practical development of the submarine periscope.

To deal with the whole subject of periscopes in one lecture was obviously impossible, and the author has accordingly limited himself to one aspect of the question, namely, the tracing of essential rays through the principal elements of the periscope, and the best arrangement of these elements.

Although calculations similar to those so comprehensively elaborated by Dr Gleichen are frequently made by designers, the system described is not the only one, and indeed in the design of longer periscopes than those made in Germany, the required results can, I think, be perhaps more easily obtained in other ways. In the first design of large periscopes in which no field lenses are placed in the focal planes, I personally prefer to consider primarily the inclinations of the principal rays and the angular apertures of the various elements, knowing by experience what angular apertures can be employed under the various conditions without unduly increasing the difficulties of the optical computation.

The work can be greatly facilitated by drawing upon the board an actual

diagram of the rays which I usually make full size. From such a diagram it is easy to decide what adjustment is required and what is the best compromise, if compromise is permissible. This method is more mechanical than Dr Gleichen's, which is essentially mathematical.

Many interesting problems of design arise when one has to consider the introduction of two powers, the so-called bifocal periscope and sky-searching arrangements, or the introduction of means for estimating the distance of a target of which some dimension is known, and these requirements make the calculation more complex.

But the design of a submarine periscope is very largely determined by the specification of the diameter and length of the top tube and the dimensions of the transition or conical portion. Indeed the first and most important stage in the design is the preliminary discussion of the conditions with the client.

As is customary in Germany and in other continental countries, Dr Gleichen expresses the illumination in terms of the square of the exit pupil diameter. The illumination is, however, a function not only of the pupil area, but also of the intensity of the light. As this intensity varies greatly it is evident that the German system is most unsatisfactory, and I should regret its adoption in this country.

The author has referred to the question of reduced illumination at the side of the field, the extent of which has so great an effect upon the design of the periscope. Although a difference of 1 per cent. in illumination can be detected in two closely contrasted fields, a gradual transition of 50 per cent. from the centre to the edge of one field cannot be detected by an observer. If the variation is greater than 50 per cent. the difference becomes observable in failing light.

It might be thought that objects could be brought into the centre of the field for particular observation, but in a submarine there is a great tactical advantage in having good illumination and definition at the margins, and good flatness of field, which features enable objects to be picked up immediately they enter the field of view.

To provide the largest usable exit pupil is also desirable, provided the periscope top is not thereby made too large and conspicuous. An exit pupil of 8 mm. which is sometimes specified is, in my opinion, unnecessarily large. It is true the human pupil opens out in the dark sufficiently to pass a beam of this diameter, but when using the periscope the retina is comparatively flooded with light, and the pupil probably cuts off a proportion of an 8 mm. beam.

It has been a great pleasure to follow Dr Gleichen's treatment of this important subject, and I wish to express my appreciation of this addition to the long and valuable series of optical papers that have been published by him.

Commander R. B. Darke agreed with Dr French that there should be good illumination at the edge as well as at the centre of the field.

The small top is not so important as it was once thought to be, for it is more difficult to see a submarine periscope than was imagined.

The bifocal type of instrument was a great boon to naval men. When the Americans entered the war, their submarines were only provided with unifocal

periscopes, and the American officers were amazed at the superiority of the British instruments.

In some of the periscopes taken from the surrendered German submarines the glasses appeared to be clearer than in some of ours which showed a yellowish tinge.

Dr French said that might be true in the earlier ones, but after 1915 the German instruments exhibited a marked deterioration. The yellowish tinge referred to was partly attributable to the clay available during the war for making the pots for glass manufacture. He had recently tried to obtain clay from Germany, but was told he would have to pay the 50 per cent. tax. He had, however, obtained from France two kinds of clay which appeared to be quite as good as the German type.

Commander Darke, continuing, pointed out that from the submarine point of view it is naturally desirable to keep the diameter of the main periscope tube as small as possible to avoid wash. On the other hand, if the main tube is small, vibration very seriously affects the use of the periscope. The vibration even in the case of a 6-inch periscope is great if the submarine is travelling at more than 6 or 7 knots. It is therefore necessary to affect a compromise in the size of the periscope which will combine the maximum optical efficiency with the minimum amount of vibration. To do this of course increases the difficulties with which the periscope maker has to contend.

Mr T. Smith said it was of interest to see how Dr Gleichen approached the problem, and presumed that the paper represented the method by which his results had been reached. It appeared to him that a treatment which disregarded detailed considerations of the lenses was simpler and naturally led to the discussion of questions which Dr Gleichen had neglected. Such a method was outlined, and a more detailed discussion deferred for a separate communication.

Professor Cheshire exhibited the tops of three captured German submarine periscopes, all of the bifocal type, with sky searchers.

Instructor-Commander T. Y. Baker (partly communicated): The problem of the periscope is one that has all over the world actively engaged the attention of armies and navies and of instrument makers who supply them with material. Dr Gleichen is one of the foremost of periscope designers and the Society is to be congratulated on receiving this very able contribution from him. Within certain limits which he has set himself, and to which I will refer later, he has attacked the problem with the thoroughness characteristic of his nation and anyone desirous of constructing a periscope upon these lines will find in this paper full information for the determination of focal lengths, etc. of lenses from such data as magnification, size of exit pupil, and so on.

But although Dr Gleichen has considered in such detail the proper design of a periscope based upon certain principles of construction I could wish that he had seen fit to deal with the problem right from the beginning and shown us that these are the correct principles to adopt. I cannot help feeling that he has been guided to them rather from the standpoint of established practice than from any *ab initio* consideration of their value. The original design of a periscope was undoubtedly the alignment of two astronomical telescopes, object-glass to object-glass. The

instrument as a whole gave a telescope of unit magnification if the two separate telescopes had equal power, but could be given a different magnification by varying the relative powers of the two. Inasmuch as an astronomical telescope has an exit pupil very near the eye lens and the two telescope object-glasses are close together it follows that the exit pupil of the whole instrument must also be near the final lens of the second telescope, that is to say, in an eminently suitable place.

As the demands for longer periscopes became felt the designer found himself in difficulties. He could not increase the overall length of each telescope indefinitely without increasing the diameters, but he managed to gain length by separating the two telescopes. This retained full illumination at the centre of the field, but cut it down at the edges and the process of lengthening could be continued until the cutting down became too marked to be bearable. The next stage was to include two more astronomical telescopes and thus we have nowadays periscopes with two storeys or with four storeys.

In Dr Gleichen's paper we get a full discussion of what can be done with the two-storey periscope. Obviously this process of piling storey upon storey could go on indefinitely if it were not for the very great cutting down of light by surface reflection. There is normally about 5 per cent. loss of light per glass-air surface and about 1 per cent. absorption loss per centimetre of axial thickness. Now simple astronomical telescopes such as are considered in this paper have three lenses so that a two-storey periscope must have at least twelve glass-air surfaces and a four-storey periscope twenty-four. To these numbers must be added the entrance window, the two reflectors, possibly a glass diaphragm, and an exit window. It will be seen, therefore, that a two-storey periscope must have twenty glass-air surfaces and a four-storey periscope thirty-two at least. I have certainly seen one Goerz design with thirty-eight glass-air surfaces in it. The percentage transmission through twenty surfaces is about 36 per cent. and through thirty-two about 19 per cent., excluding the loss by absorption that will reduce these figures to 33 per cent. and 17 per cent. approximately.

These figures refer to the centre of the field only. At the edge they will be much reduced. If we take $\mu = \frac{1}{2}$ the edge illumination is only 39 per cent. of the central illumination. Owing to this falling off the total amount of light received by the eye is only 22 per cent. of what enters the two-storey periscope and only 11 per cent. of what enters the four-storey instrument. The figures must be reduced still further to allow the two end windows being normally imperfectly clean.

In view of this very great loss of light it obviously becomes an imperative point in the design of a periscope that the minimum number of lenses should be used. Does this two-telescope design fulfil that condition? As far as I know there is no proof that it does. The fact that periscopes are designed in this way is much more probably due to its being a convenient system for the designer rather than to its being the theoretically best design obtainable. If we consider the periscope simply as a very long terrestrial telescope with a low magnification, there is no inherent reason why it should require six lenses; ordinarily the terrestrial telescope requires but five.

I should like to see the whole subject of periscope design started afresh somewhat on the following lines:

In the first place designs should be worked out without reference to the reduced top. For a tube of given bore, for a given size of exit pupil, magnification, and field of view there is a definite upper limit to the length of telescope, astronomical or otherwise, that can be made with say three lenses. We obtain a bigger length with four lenses, a bigger one again with five, and so on. In this way it should be perfectly possible to obtain maximum lengths of periscopes for uniform bores of tube—say four or five inches up to any number of component lenses decided upon. In each one of these cases the reduced top and conical shoulder can be afterwards allowed for. They will reduce length in each case and we shall thus obtain a series of steps for periscope lengths based upon the number of lenses used rather than upon the number of storeys. I feel perfectly confident that if a series of purely Gaussian designs of this nature were got out we should find that it was possible to obtain a considerable reduction in the number of lenses required for a periscope of given length.

Dr J. S. Anderson (communicated): I can endorse Dr French's statement regarding the deterioration of German instruments after about 1915. I had the privilege of dissecting a number of captured German optical instruments during the war and it was easy to detect in the later instruments a decided falling off in general workmanship, both mechanical and optical.

I am glad that Dr French objects to the continental system of specifying illumination. It should surely be a comparatively easy matter to define it in terms of the factors which govern its magnitude, namely, the area of the exit pupil, the number of glass-air surfaces in the instrument, and the intensity of the light. The theoretical specification would then be a nearer approximation, in any given case, to the actual value as determined by photometrical measurements.

The President expressed the hope that Dr French would see his way in the near future to give the Society details of British work on periscopes.

THE INTEROCULAR DISTANCE

By JAMES WEIR FRENCH, D.Sc.

MS. received, 19th October, 1921. Read and discussed, 10th November, 1921.

SYNOPSIS. Particulars of individuals tested. Historical remarks. Means employed. Mean and extreme values for men and women. Application of results to Galilean and prismatic binoculars. Variation of distance with width of head. Variation with height. Variation with age. Width of eye in relation to age. Colours of iris. Diameter of iris. Abnormalities.

WHEN in Germany recently I was told by an amiable German officer, with a keen sense of humour, that German heads, being broader than British ones, their breadth of view was greater and, their interocular distance being greater, they were able to see events in better perspective.

Being personally interested in the question of binocular vision, I decided to determine a British value of the average interocular distance. I accordingly took the requisite measurements of 409 people. As the average value had become stable long before this number was reached, it was not considered necessary to deal with still larger numbers. Of the total, 78 were male apprentices whose ages ranged from 15 to 20 years, 52 were girls of from 20 to 35 years, whose ages as recorded may not, however, be scientifically exact, and the remaining 279 were men whose ages varied from 21 to 50.

About 13 per cent. of the total were English and 6.1 per cent. Irish. The remainder were Scotch. There were no Welshmen. Among the men, exclusive of apprentices, the proportion of Englishmen was somewhat greater, being 16 per cent. Considering the large proportion of Scotch, the British character of the results may be questioned south of the Border. If so, I hope that someone in England may find it convenient to complete the observations. Personally I do not anticipate the results from north and south will differ greatly.

In deriving the mean values it is desirable, I think, to exclude the younger apprentices of 15, 16 and 17 years who may not be fully developed. It should be remarked that the older apprentices in question were exceptionally well developed, owing to the particular circumstances being such that from among the many applicants for apprenticeship it has been possible to select youths who are above the average, not only as regards education, but also as regards physical development. Most of the older apprentices may be regarded as adults.

Moritz von Rohr, in his work entitled *Die Binokularen Instrumente*, represents the extreme values of the interocular distance as being 50 to 72 mm. But he makes no reference to the age or stage of development of the lower limit individuals tested. An interocular distance of 50 mm. in the case of an adult seems very small indeed. In my measurements the lowest values recorded were 54 mm. in the case of an

adult woman; 56 mm. in the case of an apprentice aged 15, who presumably is not fully developed; 57 mm. in the case of several adult men of varying ages, one apprentice aged 15, and two apprentices aged 16. As regards the higher limit, that indicated by von Rohr, namely 72 mm., is identical with the highest value obtained by me. These results do not support the German claim referred to in the opening paragraph of this paper.

There are not many records of early interocular distance measurements. For the history of the subject one naturally turns to the works of Father Cherubin, *La Dioptrique Oculaire* of 1671, and *La Vision Parfaite* of 1677, and to Johannes Zahn's *Oculus Artificialis Teledioptricus* of 1702. According to the latter author, probably the earliest measurements recorded are those of Antonius Maria Schrylaeus de Rheita* whose own ocular separation was $\frac{23}{10}$ of a Roman foot, that is, 2.56 inches or 65 mm. For adult males the average value obtained from my measurements is 63 mm., which is only $\frac{1}{10}$ inch less than $2\frac{1}{2}$ inches.

Father Cherubin† describes fully not only his method of making the measurements by means of a pair of compasses and a looking-glass, but describes the errors of observation that may easily occur if the eyes are converged upon a near object. His own interocular distance is given as being exactly $2\frac{1}{2}$ inches, that is, about 63.5 mm., which is practically the mean value of my results. In *La Vision Parfaite* Cherubin describes and illustrates other apparatus for measuring ocular separations.

According to the *Bulletin* No. 27 of the Bureau of Standards, the limits specified are 58 to 72 mm. These figures certainly refer to adults. The arithmetical mean is 65 mm., which appears to be rather great. On the other hand, the arithmetical mean of von Rohr's extreme values, namely 50 to 72, is 61 mm., which is too small, owing doubtless to the inclusion of measurements of unduly young people.

It is interesting that in all the cases cited the maximum value recorded does not exceed 72 mm. Kepler, however, in discussing the question of ocular convergence, based his calculations upon a distance of one palmus, that is, 74 mm. This length of one Roman palmus was evidently chosen as being a convenient round figure. It is improbable that it was obtained by measurement, or that it represented the interocular distance of Kepler himself.

For the measurement of the interocular distance I made use of a small Zeiss type pupilometer, which is a simple and convenient piece of apparatus akin to that used by Schrylaeus de Rheita and Father Cherubin, but with a scale mounted below the eyes in place of the compass points.

Notwithstanding the simplicity of the instrument, several individuals were at first unable to make correct observations with it, owing probably to nervousness. In all cases check measurements were made by a separate observer by means of a pair of compasses having ball points about 3 mm. in diameter. While the individual looked at a distant object, the compasses were raised from below until they were just below the pupils and then adjusted to the distance. As Father Cherubin remarked, there is a great tendency for the individual to converge his eyes upon a near object, particularly if the compass points come into his line of vision, in which

* *Oculo Enoch et Eliae.*

† *La Dioptrique Oculaire*, Sect. IV, Chap. i, p. 204.

case the pupil separation is less than the maximum corresponding with the centres of rotation of the eyes.

The measurements were continued when necessary until there was agreement between the pupilometer readings taken by the individual himself and the compass readings taken by the separate observer.

Curve *A* in Fig. 1 is plotted from the measurements obtained from the men of 18 years and over. Abscissae represent interocular distances in millimetres and ordinates the numbers of men in each particular group of interocular distances, taken in steps of 1 mm., although the measurements were made to within 0.5 mm. If the space enclosed by the curve is bisected, it will be seen that the mean value is

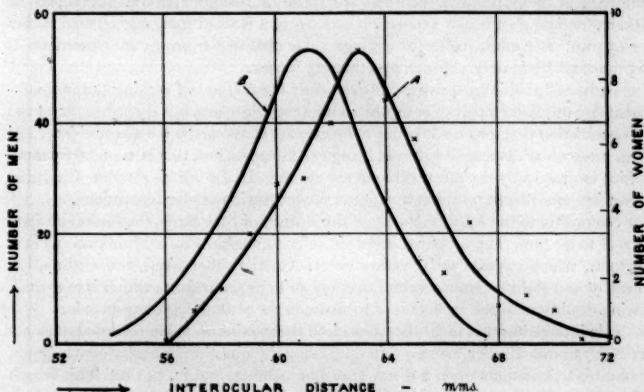


Fig. 1

63 mm., that is, 2.5 inches to within $\frac{1}{100}$ of an inch. The maximum ordinate is also 63 mm., which shows that the areas on either side of it are equal.

Commencing at the smallest value of 56 mm., the curve rises fairly regularly, but at the opposite end the curve flattens out rapidly towards the maximum value, suggesting the possible existence of abnormal cases having a greater separation than 72 mm. It will be observed that the numbers who have the greater widths are less than those who have the smaller. Thus, while there are only four individuals in the 70 mm. group, which is 2 mm. less than the maximum, there are ten in the 58 mm. group, which is 2 mm. greater than the minimum.

The curve *B* of Fig. 1 is plotted from the results obtained from the 50 women. Although the numbers involved are comparatively small, the character of the curve is very much that of the previous one. For purposes of comparison the scale of ordinates of curve *B* indicated at the right-hand side has been made greater than that of curve *A* by an amount sufficient to make the maximum ordinates equal.

It will be observed that the whole curve *B* is displaced towards the left or shorter separation side of the men's curve, which is understandable in view of the smaller average build of women as compared with men. As in the men's curve the maximum ordinate also bisects the area contained within the curve. Its value is 61 mm. and therefore the average interocular distance of the women is 2 mm. less than that of the men.

The smallest separation recorded in the case of the women was 54 mm., that is, 2 mm. smaller than the smallest male value. The maximum separation was 67.5 mm., that is, 4.5 mm. smaller than the highest male value.

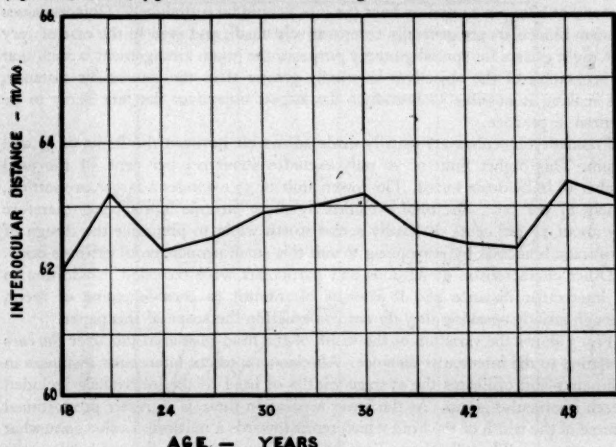


Fig. 2

For adults of both sexes the average interocular distance is 62 mm., for adult men 63 mm., and for adult women 61 mm.

After seventeen years of age there does not appear to be any very definite change in the average interocular distance of the male adults, as will be evident from Fig. 2. Abscissae represent ages and ordinates the average separation for each age group. At fifteen years the average was 59.6 mm. and at sixteen years 61.4 mm. These points have been omitted in Fig. 2. The first point on the curve represents the average for the three years, 16, 17 and 18, and succeeding points are for groups of three years, as the curve plotted on the basis of single years becomes erratic towards the right end, owing to the smaller number of the individuals in the older groups. In general, it would appear that the average value has not appreciably altered with age after the seventeenth or eighteenth year.

All prismatic binoculars can be adjusted for interocular distance and a similar

adjustment is now often provided in the case of Galilean glasses and particularly those of higher powers. Large numbers of adjustable Galilean glasses were used by the German field forces.

A glass that is adjustable between the limits of 56 mm. and 72 mm. would suit most users with the exception of a few younger people and particularly women. In the case of a Galilean glass the lower value of 56 mm. places a limit on the daylight diameter of the objective which could not exceed 45 mm., that is, 21 lines, allowing 5.5 mm. for the thickness of the cell, tube, and sunshade. If the binocular is designed for the average separation of 63 mm., then objectives having a daylight diameter of between 24 and 25 lines (52 mm.) could be introduced. Object-glasses of prism binoculars are generally comparatively small, and even in the case of very large night glasses for special gunnery purposes the prism arrangement is such that the separation of the objectives is usually greater than the interocular distance, thus making it possible to introduce the largest objectives that are likely to be required in practice.

Prismatic binoculars are usually made adjustable between the limits of 57 and 70 mm. This higher limit of 70 only excludes about 0.5 per cent. of the total number of individuals tested. The lower limit of 57 excludes a larger proportion, namely $1\frac{1}{4}$ per cent., the total excluded by these extreme limits being therefore only about $1\frac{1}{2}$ per cent. It hardly seems worth while to prejudice the design of a prismatic binocular by attempting to suit this small proportion of extreme cases.

Other characteristic dimensions and particulars were recorded in addition to the interocular distance and it may be of interest to consider some of them, although strictly speaking they do not come within the scope of this paper.

Fig. 3 shows the variation of the width of the head measured just over the ears in relation to the interocular distance. Abscissae represent interocular distances in millimetres and ordinates the average widths of head of the individuals included in each interocular group. At the lower separation there is a greater proportional increase of the width of the head which tends towards a maximum value somewhat below 160 mm. Thus the larger interocular distances generally result from the eyes being set nearer the sides of the face rather than from a proportional increase of the lateral dimensions of the head and associated parts.

That there may be very abnormal cases is evident from the point *A*. It represents an individual having a head 168 mm. wide with an interocular distance of 72 mm., whereas, according to the curve, this interocular distance should correspond with a head of about 10 mm. less width.

As is to be expected, there is a fairly definite relationship between the interocular distance and the height of normally developed individuals, as is indicated by Fig. 4. The more rapid rise at the right end of the curve is probably due to the small number of very tall people among those tested.

A curve less easily understood is that represented in Fig. 5 (curve *A*). It shows the variation with age of the width of the exposed portion of the eye. Abscissae represent ages and ordinates widths of eye. At thirty years of age the average curve *B* rapidly falls and at forty-five years tends to become level again. The

decrease in width appears to be due to the sinking of the eyeball within its orbital cavity, arising possibly from a shrinkage of the adipose tissue lining the cavity. That the eyes should sink in old age is to be expected. It is somewhat surprising to find so early a commencement of this appearance as the curve would seem to indicate.

No very wide variation of the iris apparent diameter was observed. The smallest diameter was 10 mm. and the largest 13 mm., and the average for all individuals tested was 12 mm.

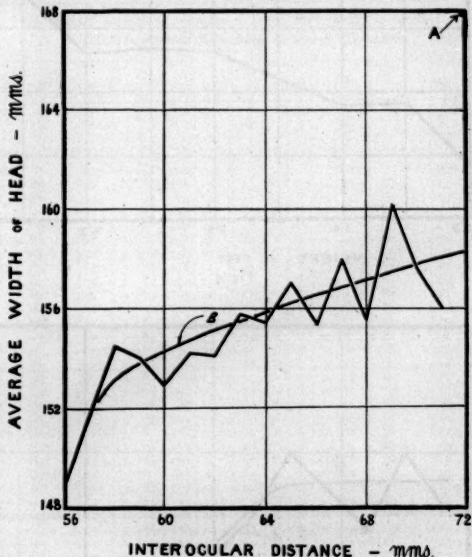


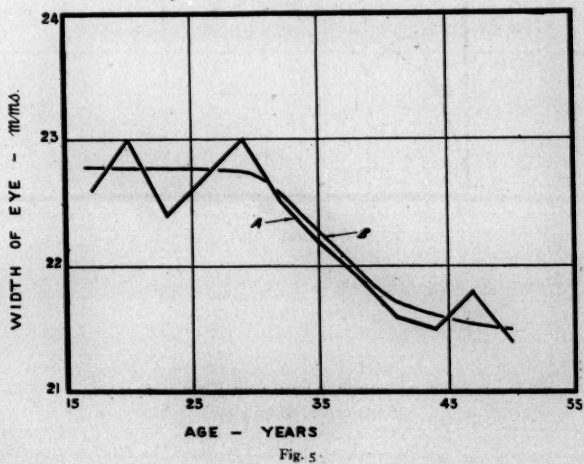
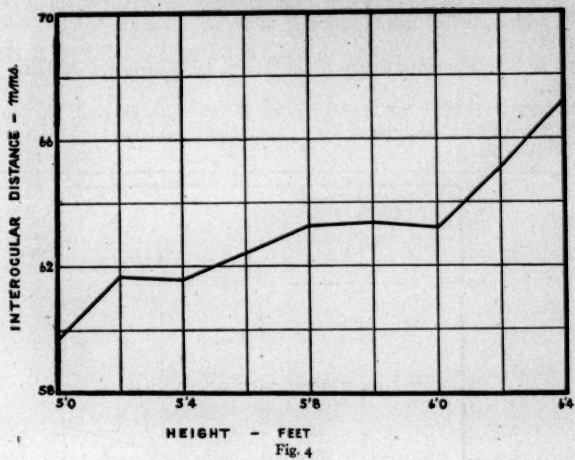
Fig. 3

As regards the colour of the iris, in the case of the men tested,

- 31 per cent. had brown eyes,
- 30 per cent. grey eyes, and
- 24 per cent. blue eyes.

In the case of the women, the most popular colour was grey,

- 46 per cent. had grey eyes,
- 27 per cent. brown eyes, and
- 23 per cent. blue eyes.



The Interocular Distance

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Thus, as compared with the men, the positions of grey and brown have been reversed but blue comes last in both cases. An equal percentage, namely 3·8 per cent., of men and women had greenish-grey eyes.

The two eyes of four men differed in colour. One man had a smaller right iris than left, and one man had a smaller left than right. Other dimensions were recorded but, as they do not exhibit any well-defined relationships, they need not be considered on this occasion.

The data from which the illustrations have been compiled are annexed in tabular form.

M=male, F=female, E=English, I=Irish, Mx=Manx, S=Scotch,

Bl=blue, Br=brown, Gn=green, Gy=grey, H=hazel.

Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye	Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye
				mm.		mm.	ft. ins.	mm.	mm.					mm.		mm.	ft. ins.	mm.	mm.
1	M	S	20	72	Br	12	6 1/2	168	25	43	M	S	34	66·5	Br	12·5	6 1/2	158	23
2	M	S	22	60	Gy	12	5 9/16	149	15	44	M	S	34	62	Gy	12	5 7/16	160	24
3	M	S	25	65	Br	12	5 7/16	148	24	45	M	S	30	68	Br	12	5 10/16	157	23
4	M	S	20	65	Bl-Gy	13	5 8	160	23	46	M	E	32	61	Gy	11·5	5 10/16	149	23
5	M	S	25	61	Bl	12	5 9	151	25	47	M	S	35	64·5	Br	12	5 7/16	165	23·5
6	M	S	21	61	Br	12	5 10/16	159	23	48	M	S	32	59	Gy	12	5 9	160	22
7	M	S	21	63·5	Br	12	5 4 1/2	156	24	49	M	E	33	58·5	Br	12	5 2 1/2	154	22
8	M	S	23	61	Br	12	6 1	160	23	50	M	S	33	59·5	Gy	12	5 4 1/2	143	21
9	M	S	24	63·5	Br	12	5 7/16	163	24	51	M	I	31	63·5	Br	12	5 5 1/2	150	22
10	M	S	23	61·5	H	12	5 9	152	23	52	M	S	35	66	Br	11	5 4 1/2	157	20
11	M	S	20	67	Bl	12	5 6	154	25·5	53	M	S	34	61	Gy	12	6 0 1/2	160	22
12	M	S	22	63	Gy-Gn	12	5 6 1/2	146	23	54	M	S	35	66	Bl	12	5 7 1/2	157	21·5
13	M	S	24	60	H	11·5	5 10	160	21·5	55	M	E	33	64·5	Br	11·5	5 7 1/2	156	23
14	M	S	20	61	Bl	11·5	5 6 1/2	148	22·5	56	M	E	35	60	Gy	11	6 0 1/2	157	22
15	M	S	25	60	Gy	10·5	5 8 1/2	151	23·5	57	M	S	30	65	H	12	6 0	159	24
16	M	S	23	60·5	Gy	11·5	5 5 1/2	141	24	58	M	S	34	66	Gy	12	5 7	160	24
17	M	S	20	62	Gy	12	5 5 1/2	148	23	59	M	S	30	64	Br	12	5 11 1/2	156	23
18	M	S	22	63·5	Gy	12	5 8 1/2	154	24	60	M	S	35	66·5	Br	12	5 8	161	23
19	M	S	21	61·5	Bl-Gy	11·5	5 8	152	23	61	M	S	33	66	Bl	11·5	5 10	154	23
20	M	S	24	61	Br	11·5	5 5 1/2	152	23	62	M	S	37	65	Bl-Gy	12	6 0 1/2	156	22
21	M	S	20	60	Br	11·5	5 6 1/2	151	22	63	M	I	36	58	Bl	12	5 6 1/2	152	21
22	M	S	28	57	Gy-Br	12	5 10	148	22	64	M	S	37	69	Gy	12	5 4 1/2	158	23
23	M	S	27	57	Br	11	5 8 1/2	151	21	65	M	S	36	57·5	Gy	12·5	5 4 1/2	151	21·5
24	M	S	27	60·5	Gy-Br	12	5 6 1/2	155	23·5	66	M	S	39	58	Br	12	5 6	150	22
25	M	S	29	62	Gy-Br	12	5 5 1/2	155	24·5	67	M	S	38	63	Gy-Bl	11·5	5 4 1/2	158	20
26	M	S	28	64	Gy	13	5 5 1/2	156	24	68	M	S	36	65·5	Gy	12	5 12	156	22·5
27	M	S	27	65	Bl	12	5 11	166	23	69	M	E	37	62·5	Gy	11·5	5 6 1/2	153	23·5
28	M	I	30	63·5	Br	12	5 8	156	22	70	M	S	36	67	Gy	12	5 4 1/2	150	21·5
29	M	S	27	58	Br	12	5 6 1/2	158	26	71	M	S	40	60·5	Br	12	5 4 1/2	150	22
30	M	S	29	60·5	Br	11	5 8 1/2	156	22	72	M	S	40	59	Gy	12	5 10 1/2	157	21·5
31	M	S	26	58	Gy	12	5 5 1/2	160	22·5	73	M	S	39	63·5	Br	12	5 7 1/2	156	22
32	M	S	28	63·5	Br	12	5 6	154	23·5	74	M	I	35	67	Bl	11·5	5 8 1/2	153	23
33	M	S	26	66·5	Br	12	5 6 1/2	159	25	75	M	S	38	58	H	12	5 5 1/2	154	20·5
34	M	S	29	65	Br	12	6 0	147	23	76	M	S	38	63	Gy	11·5	5 6 1/2	158	21
35	M	S	30	69·5	Br	13	5 7 1/2	151	26·5	77	M	S	38	63	Gy	12	5 5 1/2	155	22
36	M	E	26	61	Bl	12	5 5 1/2	158	21·5	78	M	S	36	61	Gy	11	5 2 1/2	154	20·5
37	M	E	26	59·5	Bl	11·5	5 4 1/2	148	22·5	79	M	I	39	62	Gy	12·5	5 6 1/2	141	22
38	M	S	30	57	Br	11·5	5 3	144	20·5	80	M	S	40	65·5	Gy	12	5 8	156	21
39	M	S	26	63	Gy	11·5	5 7	154	22	81	M	S	38	65	Br	11·5	5 6 1/2	162	23
40	M	S	30	60	Br	11	5 4	155	21	82	M	E	44	64	Gy	12·5	5 8 1/2	149	24
41	M	S	34	63·5	Br	12	5 3	150	23	83	M	I	43	57	Gy	10·5	5 6	150	21
42	M	S	34	61·5	Gy	12	5 5	162	22·5	84	M	S	43	58	Br	12	5 5 1/2	158	22·5

Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye	Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye
				mm.		mm.	ft. ins.	mm.	mm.					mm.		mm.	ft. ins.	mm.	mm.
85	M	S	41	57.5	Br	10	5 1 1/2	161	21.5	144	M	S	26	67	Br	11	5 6 1/2	155	23
86	M	S	44	59.5	Gy	11	5 7 1/4	152	22.5	145	M	S	27	64	Gn-Gy	12	5 6 1/2	150	24
87	M	S	41	60	Br (L) (Gy (R))	11.5	5 3 1/4	159	20	146	M	S	30	62.5	Gn-Br	12	5 7 1/2	164	23
88	M	S	41	63.5	Gy	12	5 8 1/2	162	21.5	147	M	S	28	59.5	Bl	12.5	6 0 1/2	153	22.5
89	M	S	41	58.5	Br	11.5	5 3 1/4	154	23.5	148	M	S	29	62	Br	11.5	5 4 1/2	154	22
90	M	S	45	58.5	Bl	12	5 5 1/2	159	21	149	M	S	27	64	Gy	11.5	5 7 1/2	150	23
91	M	S	45	59.5	Gy	11.5	5 3 1/4	154	21	150	M	S	30	62	Gy	12	6 3	147	25
92	M	S	42	64.5	Bl	12	5 7 1/2	158	22	151	M	S	30	63	Gy	12	5 7 1/2	152	22.5
93	M	S	42	63.5	Bl	12.5	5 4 1/2	153	22.5	152	M	S	27	63.5	Bl-Gy	12.5	5 8 1/2	152	22
94	M	S	43	64	Bl	12	5 4 1/2	156	22	153	M	S	30	64	Gn-Br	12.5	5 8 1/2	158	23
95	M	S	43	61	Gy	11.5	5 10 1/2	153	23	154	M	S	27	69	Br	11.5	5 9 1/2	161	23
96	M	S	43	65	Bl	11.5	5 8 1/2	155	22	155	M	S	28	62	Br	12.5	5 7 1/2	158	24
97	M	E	42	59	Gy	12	5 8 1/2	155	21.5	156	M	S	29	65	Br	12.5	5 8 1/2	154	22.5
98	M	S	43	63.5	Br	12	5 5	160	21.5	157	M	S	27	63.5	Bl	11.5	5 8	155	21
99	M	E	42	62.5	Gy	12	5 9 1/2	155	21.5	158	M	S	30	65	Br	12.5	5 4 1/2	152	25
100	M	S	41	63.5	Gy	12	5 5	159	19.5	159	M	S	26	61	Gy-Bl	11.5	5 5 1/2	157	21
101	M	S	44	59	Br	11.5	5 2 1/2	146	19.5	160	M	S	60	68.5	Gy	12	5 6	159	23
102	M	S	43	65	Bl	12.5	5 7	160	22	161	M	S	27	63	Gy	12	5 8 1/2	152	22.5
103	M	S	47	67	Gy	12	5 7	154	21.5	162	M	S	35	62.5	Gy	12	5 5	157	21.5
104	M	E	50	63.5	Gy	12	5 9 1/2	165	21.5	163	M	S	31	61.5	Bl	11	5 7 1/2	154	23
105	M	E	46	64.5	Gy	12	5 10 1/2	151	22	164	M	S	33	64.5	Bl	12	5 5 1/2	158	23.5
106	M	S	49	68	Bl	12	5 8 1/2	153	22.5	165	M	S	35	64	Gy	11.5	5 5 1/2	154	23.5
107	M	S	48	65	Bl	12	5 8 1/2	153	23	166	M	E	31	61	Br	11	5 7	149	22
108	M	S	48	63.5	Bl	11.5	5 5 1/2	160	23	167	M	S	31	63	Gy	12	5 9 1/2	158	22.5
109	M	S	49	63	Gy	12	5 2	156	21.5	168	M	S	33	66	Bl	12	4 11 1/2	148	23
110	M	S	49	60	Br	12	5 9 1/2	157	21	169	M	I	35	63	H	12.5	5 4 1/2	153	23.5
111	M	S	49	64.5	Gy	11	5 2 1/2	155	22	170	M	S	34	62	Gy	12	5 7 1/2	158	22.5
112	M	S	50	60.5	Br	12	5 4	161	22	171	M	S	32	59	Bl	12.5	5 4 1/2	150	24
113	M	S	46	63	Gy	12	5 6	150	22	172	M	S	31	63	Gy	12	5 8 1/2	165	23
114	M	S	50	62	Gy	11	5 9 1/2	157	21	173	M	E	31	67	Bl-Gn	12	5 5 1/2	164	23
115	M	S	47	64	Gy	12	5 5 1/2	160	22.5	174	M	S	34	65	Br	12	5 8 1/2	158	23.5
116	M	S	48	59.5	Bl	11.5	5 6 1/2	161	21	175	M	S	31	66.5	Gy	11.5	5 2 1/2	160	24
117	M	S	47	63	Bl-Gy	10.5	5 0 1/2	159	20	176	M	I	32	66	Bl	12.5	5 9	155	22
118	M	S	48	65	Bl	11.5	5 9 1/2	167	22.5	177	M	E	31	63.5	Bl	12	5 10	154	24
119	M	S	50	61	Bl	11.5	5 8 1/2	157	21	178	M	S	34	61	Bl	12	5 2 1/2	162	23
120	M	S	50	69	Br	12	5 5	161	22	179	M	I	31	63.5	Gy	11	5 6	154	21
121	M	S	47	59	Gy	12	5 6 1/2	165	21	180	M	S	35	63	Bl	12	5 10 1/2	161	22
122	M	S	24	59.5	Br	11	5 9 1/2	160	21	181	M	S	40	61.5	Gy-Br	11	5 8 1/2	158	21.5
123	M	S	23	61	Br	12	5 6 1/2	148	22	182	M	S	36	62.5	Gy	11.5	5 6 1/2	163	23
124	M	S	22	58.5	Gy	11	5 9 1/2	150	21	183	M	S	38	62.5	Br	12.5	5 7 1/2	156	23.5
125	M	I	25	64	Br	11.5	5 5 1/2	160	21	184	M	S	36	70	Bl	11.5	5 5 1/2	150	20
126	M	S	24	62.5	Bl	12	5 10	159	22.5	185	M	S	35	62.5	Gy	11.5	5 5 1/2	154	21
127	M	S	21	58.5	Gy	12.5	5 11 1/2	149	22.5	186	M	S	36	70	Bl	11.5	5 9	156	23
128	M	S	20	67.5	Gy	12	5 9 1/2	154	24	187	M	S	37	67.5	Bl	12.5	5 5 1/2	153	21.5
129	M	S	21	58.5	Br	11.5	5 11 1/2	153	20.5	188	M	S	40	62.5	Bl	12	5 8 1/2	154	22.5
130	M	S	21	67	Gy	12.5	5 9 1/2	162	22	189	M	S	37	67	Gn-Gy	12	5 3	158	23.5
131	M	S	22	61	Gy	11	5 4 1/2	161	21	190	M	S	36	62.5	H	12.5	5 8 1/2	156	22
132	M	S	25	63.5	Br	12	5 6 1/2	151	21	191	M	E	36	63	Gy	12	5 8 1/2	158	22.5
133	M	S	20	63.5	Br	12	5 5 1/2	152	22	192	M	S	35	60	Bl	12.5	5 8	160	22
134	M	S	24	61.5	Br	12	5 8 1/2	150	22.5	193	M	S	39	57	Bl	11.5	5 5 1/2	155	19
135	M	S	25	58.5	Bl	12	5 5 1/2	145	22.5	194	M	S	38	63	Bl	13	5 8 1/2	156	22.5
136	M	S	21	62.5	Gy	12	5 4 1/2	153	22	195	M	S	38	62	Bl	12	5 10	159	23
137	M	S	25	61.5	Bl	12	5 7	153	23.5	196	M	S	37	61.5	Bl	12	5 10 1/2	157	21.5
138	M	S	21	60	Bl	11.5	5 6 1/2	148	21.5	197	M	S	35	63	Gy	12	5 8	160	22
139	M	S	21	68.5	Gy-Br	11.5	5 9	157	23	198	M	S	43	57.5	Gy	12	5 3 1/2	160	22
140	M	S	20	63	Br	12.5	5 4 1/2	153	23	199	M	E	41	66	Gy	11.5	5 8 1/2	160	20.5
141	M	S	24	68.5	Bl	12	6 1 1/2	150	22.5	200	M	S	42	62.5	Gy	12	5 2 1/2	156	22
142	M	S	27	62.5	Gy-Br	12	5 5 1/2	160	23	201	M	S	45	70	Br	11	5 6 1/2	164	20
143	M	S	29	62	Bl	12	5 5	150	22	202	M	S	42	68	Gy	12.5	5 7 1/2	153	22
										203	M	S	44	61.5	Br	12	5 3 1/2	158	21

Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye	Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye
				mm.		mm.	ft. ins.	mm.	mm.					mm.		mm.	ft. ins.	mm.	mm.
204	M	S	44	65	Br	12	6 0	167	22	263	M	S	31	64	Gy	12.5	5 9	154	24.5
205	M	S	45	67	Bl	12	5 11 1/2	170	22	264	M	S	32	61.5	Br	12	5 7 1/2	152	21.5
206	M	S	45	65	Bl	12	5 9 1/4	155	23	265	M	S	33	64.5	Gy	12	5 9	160	23
207	M	S	44	63.5	Br	11	5 3 1/2	154	22	266	M	S	33	62	Bl	12	5 5 1/2	152	21
208	M	S	44	65	Gy	12.5	5 10 1/2	150	22	267	M	S	31	60.5	Br	11.5	5 7 1/2	148	23.5
209	M	S	41	65	Br	12	5 3	159	21	268	M	S	34	63	Bl	12	5 8	163	25
210	M	S	44	62	Gy	11.5	5 4	151	20	269	M	S	32	61	Br	12	5 5	137	22
211	M	S	43	63	Gy	12	5 4 1/2	159	21	270	M	S	31	62.5	Br	12	5 0 1/2	158	22
212	M	S	49	64	Gy	11.5	5 4	158	21.5	271	M	S	33	69.5	Br	12.5	5 8 1/2	156	23.5
213	M	E	40	59.5	Bl	11.5	5 2	159	20	272	M	S	33	65	Br	13	5 6	153	23
214	M	E	46	64	Gy	11	5 8 1/4	155	21.5	273	M	S	30	67	Bl	13	5 4 1/2	157	22.5
215	M	S	46	71	Bl-Gy	12.5	5 3 1/2	156	23	274	M	S	30	66	Bl	11.5	5 11 1/2	149	25
216	M	S	46	67	Gy	12	5 11	163	23	275	M	S	34	70	Gy	12	5 8 1/2	150	22
217	M	S	48	61	Gy	10.5	5 4 1/2	152	23	276	M	S	33	65	Bl	12	5 7	155	22
218	M	S	48	61.5	Br	12.5	5 6 1/2	150	21	277	M	S	33	68.5	Gy	11	6 0 1/2	162	21.5
219	M	E	46	65.5	Br	11.5	5 7 1/2	162	21.5	278	M	S	30	61	Bl	12	5 7 1/2	153	22
220	M	S	50	60	Gy	11	5 4	155	21	279	M	S	31	62	Gn-Br	11.5	5 9	153	23
221	M	I	46	60	Br	11.5	5 6 1/2	152	22	280	M	S	19	58.5	Gy	12	5 7	161	24
222	M	S	46	61.5	H	11.5	5 6	150	21	281	M	S	19	66.5	Bl	12	5 11 1/2	154	23.5
223	M	S	50	60	Bl	11.5	5 7 1/2	156	20	282	M	S	20	61	Bl	12	5 6	162	24
224	M	S	46	60.5	Br	12	5 5	156	22	283	M	E	16	57.5	Br	12.5	5 5	153	21.5
225	M	S	46	62	Bl	12.5	5 5 1/2	155	23.5	284	M	S	15	62	Gy	12.5	5 1 1/2	159	22
226	M	S	46	66.5	Br	11	5 6 1/2	150	20	285	M	S	17	62.5	Bl-Gy	13	5 11	149	24
227	M	E	48	64.5	Bl	12	5 9 1/4	155	21.5	286	M	S	15	61	Gy	13	4 11 1/2	154	24
228	M	S	21	63.5	H	11.5	5 8 1/2	150	23.5	287	M	S	17	65	Gy	12	5 11	150	23
229	M	S	22	65	Br	12	5 7	165	21	288	M	S	16	62	Gy	13	5 4 1/2	148	23
230	M	S	20	62.5	Bl	13	5 9 1/2	154	25	289	M	S	15	63.5	Gn-Bl	11.5	5 7 1/2	156	21
231	M	S	24	65	Gy-Bl	11	5 6 1/2	151	24	290	M	S	15	60	Bl	12	5 0 1/2	141	23
232	M	I	24	69.5	Br	12	5 2 1/2	166	24	291	M	S	16	62	Bl	12.5	5 3 1/2	157	23
233	M	S	20	65	Br	12.5	5 4 1/2	158	23.5	292	M	S	16	59	Br	13	5 4 1/2	156	22.5
234	M	S	20	60	Br	12	5 7	150	24	293	M	S	15	60	Gy	13	5 6 1/2	150	22
235	M	S	20	57	Bl	12	5 9 1/2	154	20.5	294	M	S	15	59	Bl	12.5	5 4 1/2	150	23
236	M	S	20	60	Bl	12	5 9 1/2	153	23	295	M	S	17	62	Gn-Gy	12.5	5 7 1/2	162	23.5
237	M	S	23	63	Bl	12	5 4 1/2	166	20.5	296	M	S	16	62	Br	12	5 8 1/2	157	24
238	M	S	20	63	Br	12	5 5 1/2	158	23	297	M	S	15	57	Br	12	5 0 1/2	160	21.5
239	M	S	20	66.5	Br	12	5 9	158	23	298	M	S	15	60	Gy	12.5	5 6 1/2	153	21
240	M	I	22	62	Gy	12	5 4 1/2	154	22	299	M	S	16	59	Br	12	5 3	154	22
241	M	S	20	65	Gy	12	5 7 1/2	148	22	300	M	S	16	60	Br	13	5 3	145	21
242	M	S	22	63	Br	12	5 6 1/2	158	23	301	M	S	16	60	Br	12	5 7	161	24.5
243	M	S	21	62	{Br(L) Bl(R)}	11	5 6 1/2	157	22	302	M	S	16	61.5	Br	12	5 11	156	22
244	M	E	30	62	Gy	12	5 8 1/2	157	23.5	303	M	S	15	59.5	Br	12	5 3	145	22
245	M	E	29	66	H	12	5 9 1/2	157	21.5	304	M	E	17	66	Br	12	5 7 1/2	160	22
246	M	E	27	61.5	Br	12	5 5 1/2	160	22.5	305	M	S	17	62.5	Gy	12.5	5 9	155	22
247	M	S	27	62.5	Bl	11.5	5 7 1/2	158	23.5	306	M	S	17	59.5	Br	12	5 5 1/2	146	22
248	M	S	29	68.5	Br	11.5	6 4	160	25	307	M	S	16	65	Br	12	5 5 1/2	151	21
249	M	I	30	62.5	Bl	11.5	5 7 1/2	150	22.5	308	M	S	16	61	Br	11	5 0 1/2	151	22
250	M	E	26	62.5	Bl	11.5	5 9 1/2	152	22.5	309	M	S	15	59.5	Gy	12.5	5 4	148	23.5
251	M	I	28	62	Bl	12	5 5 1/2	150	22	310	M	S	16	65.5	Gy	11.5	5 4 1/2	150	22
252	M	S	30	64.5	Gy	12	5 7 1/2	157	24	311	M	S	16	61	Gn-Gy	12	5 7 1/2	156	22
253	M	E	27	61.5	Br	11.5	5 10	154	20.5	312	M	S	17	62	Bl	12	5 8 1/2	158	24
254	M	S	27	62.5	Gy-Bl	11.5	5 6 1/2	155	22.5	313	M	S	15	58	Gy	11	4 10 1/2	148	19.5
255	M	S	29	63	Gy	12	5 5	149	21	314	M	S	16	62	Bl	12.5	5 10 1/2	164	24
256	M	S	26	64	Bl	12	5 8 1/2	151	22	315	M	S	15	56	Gy	11.5	5 5 1/2	153	20
257	M	S	27	62	Br	12	5 6	156	22	316	M	S	15	56	Br	11.5	5 2	149	21.5
258	M	S	26	64	Gn-Bl	12.5	5 7 1/2	150	24	317	M	S	16	62.5	Br	12.5	5 4 1/2	156	24
259	M	S	26	67	Gy	12	5 9 1/2	153	21.5	318	M	S	17	59	{Br(R) Gy(L)}	12	5 1 1/2	143	19
260	M	S	27	64	Br	11	5 5 1/2	161	24	319	M	S	16	60	Gy	12.5	5 7 1/2	153	22
261	M	S	30	67	Bl	12	5 8	162	25.5	320	M	E	16	57.5	Bl	12	6 0 1/2	149	25
262	M	Mx	31	64.5	Br	11	5 6 1/2	160	21.5	321	M	S	17	63.5	Bl	12.5	5 5 1/2	151	22

Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye	Serial no.	Sex	Nationality	Age	Separation of eyes	Colour of Iris	Diam. of Iris	Height	Width of head	Width of eye
				mm.		mm.	ft. ins.	mm.	mm.					mm.		mm.	ft. ins.	mm.	mm.
322	M	S	17	64	Gn-Bl	12.5	5 6 $\frac{1}{2}$	161	23.5	366	F	S	...	65	Gn-Gy	12	5 4 $\frac{1}{2}$	146	22
323	M	S	16	65	Br	12.5	5 0 $\frac{1}{2}$	147	23	367	F	S	...	62	Gy	12.5	5 4 $\frac{1}{2}$	140	23.5
324	M	S	17	63	Bl	12.5	5 6	151	22	368	F	S	...	61.5	Gy	12.5	5 6 $\frac{1}{2}$	133	21.5
325	M	S	17	67	Bl	12.5	5 4 $\frac{1}{2}$	158	24	369	F	S	...	60.5	Br	12.5	5 4	151	23
326	M	S	17	59	Gy	12	5 6 $\frac{1}{2}$	154	21	370	F	S	...	64	Gy	12	5 6 $\frac{1}{2}$	144	22.5
327	M	S	17	69	Bl	12	5 7	162	24	371	F	S	...	58	Bl	11.5	5 3 $\frac{1}{2}$	143	20.5
328	M	S	17	61.5	Br	12.5	5 2	148	22.5	372	F	S	...	60.5	Gy	12	5 4 $\frac{1}{2}$	148	22.5
329	M	S	18	59.5	Bl	11.5	5 5	154	21	373	F	S	...	61.5	H	11	5 6 $\frac{1}{2}$	148	19
330	M	S	17	63.5	Gy	12.5	5 6 $\frac{1}{2}$	151	24.5	374	F	S	...	59.5	Gy	12	5 3 $\frac{1}{2}$	147	23.5
331	M	E	18	61	Bl	12	5 3	160	23	375	F	S	...	61	Gy	12	5 8 $\frac{1}{2}$	157	22
332	M	S	18	59	Bl	12	5 11	148	21.5	376	F	S	...	64.5	Bl	12	5 4	143	23.5
333	M	S	19	62.5	Bl	12.5	5 7	153	23	377	F	E	...	61.5	Bl	12	5 3 $\frac{1}{2}$	146	23
334	M	S	18	63	Bl	12	5 7	155	23	378	F	E	...	55.5	Bl	12.5	5 3 $\frac{1}{2}$	143	23.5
335	M	S	18	61	Br	12.5	5 5	158	22.5	379	F	S	...	59	Bl	11.5	5 5 $\frac{1}{2}$	141	21
336	M	E	18	62.5	Br	12	5 9	154	23	380	F	S	...	61	Bl	13	5 5 $\frac{1}{2}$	152	24.5
337	M	S	18	63	Br	13	5 7	156	24	381	F	S	...	59.5	Gy	12.5	5 1 $\frac{1}{2}$	137	22
338	M	S	18	61	Br	12.5	5 8 $\frac{1}{2}$	154	23	382	F	I	...	57.5	Gy	12	5 4 $\frac{1}{2}$	152	22
339	M	S	18	59.5	Gy	11.5	5 7 $\frac{1}{2}$	154	21	383	F	S	...	63	Bl	12	5 5	152	23
340	M	S	18	59	Bl	11.5	5 9 $\frac{1}{2}$	154	23	384	F	S	...	64.5	Gn-Gy	12.5	5 5 $\frac{1}{2}$	152	24
341	M	S	18	63.5	Gy	12	5 3 $\frac{1}{2}$	154	22	385	F	S	...	58	Gy-Br	11.5	5 6	156	20.5
342	M	S	19	62.5	Bl	12	5 8	158	23	386	F	S	...	66	Gy	12.5	5 6 $\frac{1}{2}$	148	20.5
343	M	S	19	62.5	Gy	12.5	5 5	153	23	387	F	I	...	63.5	Br	12	5 7	153	23
344	M	S	19	63	Bl	12	5 10 $\frac{1}{2}$	150	24	388	F	S	...	60	Br	12	5 3 $\frac{1}{2}$	140	21.5
345	M	S	19	61.5	Br	12	5 5 $\frac{1}{2}$	156	24	389	F	E	...	63	Bl	12	5 5 $\frac{1}{2}$	155	22
346	M	S	19	62	Bl	12	5 9 $\frac{1}{2}$	154	23	390	F	S	...	57	Br	11.5	5 0 $\frac{1}{2}$	144	21
347	M	S	19	59	Br	11	5 6 $\frac{1}{2}$	155	21.5	391	F	S	...	62	Br	12	5 5	148	21
348	M	S	19	68	Br	13	5 10	162	25	392	F	S	...	60	Br	12	4 9	147	22.5
349	M	S	19	64	Gy	12	5 7	149	22.5	393	F	S	...	67	Gy	12	5 5 $\frac{1}{2}$	145	23
350	M	S	20	70	Br	12	6 2 $\frac{1}{2}$	160	24	394	F	S	...	59.5	Gy	12	5 2 $\frac{1}{2}$	151	21
351	M	S	19	64	Br	12.5	5 6	159	24	395	F	S	...	62.5	Bl	12	5 4	142	22.5
352	M	S	19	61.5	Bl	11.5	5 6 $\frac{1}{2}$	146	20.5	396	F	S	...	57.5	Gy	11.5	4 10	139	21
353	M	S	19	67	Gy	12.5	5 8	164	23	397	F	E	...	62.5	Gy	12	4 11 $\frac{1}{2}$	144	20
354	M	S	19	67	Br	12	5 9 $\frac{1}{2}$	157	24	398	F	E	...	58.5	Bl	12	5 4 $\frac{1}{2}$	146	21
355	M	S	19	60	Br	12.5	5 9 $\frac{1}{2}$	151	25	399	F	S	...	60.5	Gy	11	5 5	150	22
356	M	S	19	63.5	Br	12	6 0 $\frac{1}{2}$	156	23	400	F	S	...	62	Gy	12	5 3 $\frac{1}{2}$	151	22
357	M	S	19	64.5	H	11.5	5 6 $\frac{1}{2}$	160	23.5	401	F	S	...	58.5	Br	12	5 5 $\frac{1}{2}$	152	23
358	F	S	...	59.5	Gy	11.5	5 8 $\frac{1}{2}$	150	22	402	F	S	...	58	Br	12	5 5 $\frac{1}{2}$	156	23
359	F	S	...	60	Gy-Br	12	5 3 $\frac{1}{2}$	148	21.5	403	F	S	...	60	Gy	11.5	5 2 $\frac{1}{2}$	143	21.5
360	F	S	...	55.5	Gy	12	5 6	146	22	404	F	S	...	59	Gy	11.5	5 4 $\frac{1}{2}$	152	22
361	F	S	...	64	Bl	12	5 6	148	22	405	F	S	...	54	Gy	11.5	5 0	134	21
362	F	S	...	63.5	Gy	11.5	5 3 $\frac{1}{2}$	156	24	406	F	S	...	65	Gy	12	5 3 $\frac{1}{2}$	151	24
363	F	S	...	61.5	Gy	12	5 5 $\frac{1}{2}$	156	23	407	F	S	...	61	Br	12	5 4 $\frac{1}{2}$	148	22
364	F	S	...	60	Br	12	5 3	148	23	408	F	S	...	61	Bl	12	5 3	151	23
365	F	S	...	62	Br	11.5	5 7	155	23	409	F	S	...	62	Br	11.5	4 11	146	21

DISCUSSION

Prof. Cheshire welcomed the paper, but he thought that the results could only be accepted as true for a workshop in Glasgow. He had very little doubt that measurements made in workshops in different parts of the country would, in many cases, yield quite different results. He understood, for instance, that the Anthropometrical Society had shown that there were considerable differences in typical "County" heads, in addition to those which existed between the different races to be found in these Islands.

He thought that the paper would be welcomed by the Anthropometrical Society. He asked if the author would sketch the actual form of instrument used in the experiments.

Dr French, in reply, offered to present the instrument to the Society.

The President, in thanking the author for his kind offer, suggested that Prof. Cheshire should take charge of the instrument and make observations on the interocular distances of his pupils. He thought that it was a pity no Welshmen had been included in the series of measurements which the author had carried out, as Dr Fleure had found interesting variations in some head measurements of the Welsh.

Mr P. F. Everitt drew attention to the anthropometrical investigations carried on at the Galton Biometrical Laboratory and asked if anyone knew whether measurements of interocular distance had been made there.

Referring to Fig. 1 in the paper, he did not understand how the author could decide the form of the ends of curve *B* from the results of only 50 measurements. He thought that in curve *A* there should be some small difference between the mean value and that of the maximum.

Mr W. M. Brett asked the author if he had questioned the individuals under test as to their stereoscopic vision. He had found that a slight increase in interocular distance was often accompanied by a considerable increase in stereoscopic acuity.

The author communicated the following reply:

With regard to Mr Everitt's remarks, although the curve *B* (Fig. 1) is based only upon 50 measurements, it is fairly definite. In the case of the larger number of male observations, the general character of the curve was established at an early stage and did not vary much as the number of observations increased. The ends of the curves are certainly much more indefinite than the centres. This is unavoidable in view of the small proportion of individuals having the extreme values of the interocular distance.

In reply to Mr Brett, the stereoscopic power of the individuals was not examined. Theoretically this should be proportional to the separation of the eyes, provided other conditions are favourable. That the relationship should be observable in the variation of so short a base is, I think, remarkable, when the great variations in the other conditions are considered.

NOTE ON THE THIN ASTIGMATIC LENS

By THEODORE CHAUNDY, M.A.

MS. received, 1st June, 1921. Read, 10th November, 1921.

ABSTRACT. For the normal refraction of an aspherical wave-front at an aspherical surface three quantities are introduced which obey the Gaussian formula of symmetrical normal refraction.

THERE is a result in the first approximation theory of a thin astigmatic lens which does not seem to be so widely known as, I think, it should be. Essentially it may be stated as follows: An aspherical wave-front is refracted normally at an aspherical surface. If $1/r$, $1/s$ are the principal curvatures of the incident wave-front, their planes making angles α and $90^\circ + \alpha$ with a chosen reference-plane; if

$$1/R, 1/S, \theta, 90^\circ + \theta$$

are similar quantities for the refracting surface, and

$$1/r', 1/s', \alpha', 90^\circ + \alpha'$$

similar quantities for the emergent wave-front; then

$$u = 1/r + 1/s, u = (1/r - 1/s) \cos 2\alpha, u = (1/r - 1/s) \sin 2\alpha$$

all satisfy the Gaussian formula

$$\mu' u' - \mu u = (\mu' - \mu) U.$$

To establish this I need the analytical expression of the fundamental laws of refraction, viz.,

$$\left. \begin{aligned} \mu' l' &= \mu l + \kappa L \\ \mu' m' &= \mu m + \kappa M \\ \mu' n' &= \mu n + \kappa N \end{aligned} \right\} \dots (1).$$

Here (l, m, n) and (l', m', n') are the direction cosines of the incident and emergent rays, (L, M, N) those of the normal to the refracting surface and κ a quantity whose evaluation does not here concern us.

Now for normal refraction I may take the origin of three dimensional Cartesian coordinates as point of incidence and the x -axis as common normal. We may thus take the equation to the incident wave-front, the refracting surface and the emergent wave-front to be

$$2x = ay^2 + 2hyz + bz^2 \dots (2),$$

$$2x = Ay^2 + 2Hyz + Bz^2 \dots (3),$$

$$2x = a'y^2 + 2h'yz + b'z^2 \dots (4).$$

Consider a point (x, y, z) of the incident wave-front near the origin, i.e. y, z are small quantities of the first order and x —from equation (2)—small of the second order. The normal at (x, y, z) to (2) has direction cosines proportional to

$$1, -(ay + hz), -(hy + bz).$$

Since the sum of the squares of these quantities differs from unity by a small quantity of the second order, we may regard them as *actual* direction cosines. Any point on this normal has coordinates

$$x + \lambda, y - \lambda(ay + hz), z - \lambda(hy + bz),$$

where λ is some parameter. This cuts (3) where

$$2x + 2\lambda = Ay^2 + 2Hyz + Bz^2 + \text{quantities of higher order.}$$

Hence λ is of the second order, and to a first approximation the point of incidence on (3) is

$$(x + \lambda, y, z).$$

The normal of refraction has therefore direction cosines

$$1, -(Ay + Hz), -(Hy + Bz),$$

and the emergent ray has direction cosines

$$1, -(a'y + h'z), -(h'y + b'z).$$

Hence by equation (1)

$$\begin{aligned}\mu' &= \mu + \kappa, \\ \mu'(a'y + h'z) &= \mu(ay + hz) + \kappa(Ay + Hz), \\ \mu'(h'y + b'z) &= \mu(hy + bz) + \kappa(Hy + Bz).\end{aligned}$$

These equations are true for all small values of y, z , if (4) truly represents the emergent wave-front. Hence we may replace them by

$$\begin{aligned}\mu'a' &= \mu a + (\mu' - \mu)A \\ \mu'h' &= \mu h + (\mu' - \mu)H \\ \mu'b' &= \mu b + (\mu' - \mu)B\end{aligned} \quad \dots (5).$$

Now, since the principal curvatures of (2) are $1/r, 1/s$ lying in planes making with yOx , say, angles $\alpha, 90^\circ + \alpha$, its equation may be rewritten

$$2x = (y \cos \alpha + z \sin \alpha)^2/r + (y \sin \alpha - z \cos \alpha)^2/s,$$

so that

$$\begin{aligned}a &= \cos^2 \alpha/r + \sin^2 \alpha/s, \\ h &= \sin \alpha \cos \alpha (1/r - 1/s), \\ b &= \sin^2 \alpha/r - \cos^2 \alpha/s.\end{aligned}$$

Hence

$$\begin{aligned}a + b &= 1/r + 1/s, \\ a - b &= (1/r - 1/s) \cos 2\alpha, \\ 2h &= (1/r - 1/s) \sin 2\alpha,\end{aligned}$$

and similarly for the other surfaces.

Substitution in equation (5) gives the promised result,

$$\mu'(1/r' + 1/s') = \mu(1/r + 1/s) + (\mu' - \mu)(1/R + 1/S) \quad \dots (6),$$

$$\mu'(1/r' - 1/s') \cos 2\alpha' = \mu(1/r - 1/s) \cos 2\alpha + (\mu' - \mu)(1/R - 1/S) \cos 2\theta \quad \dots (7),$$

$$\mu'(1/r' - 1/s') \sin 2\alpha' = \mu(1/r - 1/s) \sin 2\alpha + (\mu' - \mu)(1/R - 1/S) \sin 2\theta \quad \dots (8).$$

We may replace equations (7) and (8) by the simple complex equation (using i for $\sqrt{-1}$)

$$\mu' (1/r' - 1/s') e^{2ia'} = \mu (1/r - 1/s) e^{2ia} + (\mu' - \mu) (1/R - 1/S) e^{2i\theta} \dots (9).$$

Equation (6) which shews that the mean curvature obeys the Gaussian formula is generally known, I believe. But I have no references to the corresponding property of what we may call the "components of curvature-difference," as represented by equations (7), (8). These equations represent, in point of fact, a vector-addition. That is to say, if we draw in a plane a length AB proportional to $\mu (1/r - 1/s)$ and in a direction 2α , and then a length BC proportional to $(\mu' - \mu) (1/R - 1/S)$ and in a direction 2θ , then AC is proportional to $\mu' (1/r' - 1/s')$, and is in a direction $2\alpha'$.

Finally, if we have a succession of normal refractions, occurring effectively at the same point—e.g. as occasioned by passage through a *thin* astigmatic lens or by a series of such lenses in close contact—then equations of the types of (6), (7), (8) are applicable to each refraction, and in full analogy with Gauss's results we see that the "mean powers" and the "components of power-difference" of the constituent surfaces are additive to give the "mean power" and the "components of power-difference" of the complete system.

OBITUARY NOTICE

WILLIAM SHACKLETON, F.R.A.S., A.R.C.Sc., F.INST.P.

By the death of Mr WILLIAM SHACKLETON the Optical Society has lost one of its most able and active supporters. Mr Shackleton joined the Society in 1913, and on taking office as Secretary in 1916 he spared no effort to increase its status and widen its influence. He was especially impressed by the importance of closer co-operation between the makers of instruments and men of science, and the growth of membership of both classes during his tenure of office was a source of great satisfaction to him. He contributed very materially to the progress of the Society by his constant endeavour to make the meetings attractive, and was frequently instrumental in securing matter of exceptional interest for exhibition or discussion. He was elected a Vice-President in 1920 and continued to take an active interest in the welfare of the Society.

Mr Shackleton was born at Keighley in 1871, and received his early education at the Trade and Grammar School, where he distinguished himself in scientific subjects and athletics. From 1888 to 1891 he continued his studies at the Royal College of Science, taking his final course in Physics under Sir Arthur Rucker and Prof. Boys. During the succeeding ten years he was an assistant to Sir Norman Lockyer at the Solar Physics Observatory and developed a love for astronomy and optics which was maintained throughout his life. He took a prominent part in the early work at South Kensington on the photography of stellar spectra, and the success of this work, carried on under unfavourable atmospheric conditions, was in a considerable measure due to Shackleton's readiness to go to the observatory at any hour of the night if there was the least chance of the sky becoming clear. During this period he took part in two notable Eclipse Expeditions organised by Sir Norman Lockyer. In 1893, in company with Mr Albert Taylor, he was stationed in Brazil, and obtained some interesting photographs with the then novel prismatic camera. Unfortunately, an attack of malaria which he then contracted recurred at intervals and caused him considerable discomfort and loss of time during several years. In 1896 he was in Novaya Zemlaya with Dr E. J. Stone. This expedition was made possible by the public spirit of Sir George Baden-Powell, who conveyed the observers to the eclipse station in his yacht *Otaria*. Serious delay was caused by the grounding of the yacht, and by generally unfavourable weather for preparations, but Shackleton was never lacking in resource and was able to take full advantage of a fortunate interval of clear sky at the critical moments. He achieved a well-deserved and historic success by photographing for the first time the complete "flash spectrum" at the beginning of totality.

Towards the end of 1900 Shackleton was selected to accompany Scott's first expedition to the Antarctic as physicist, and was occupied for some months with the special instruments to be employed, but almost at the last moment, to his great disappointment, he was disqualified on medical grounds.

His next appointment was with Dr A. A. Common, who was celebrated for his construction of large reflecting telescopes, but whose work at this time was mainly connected with the design and manufacture of gun-sights and range-finders. Shackleton, however, gained an insight into practical optics and business methods of which he afterwards reaped the benefit. On the death of Dr Common he returned to the Royal College of Science for a short time as a demonstrator in the Physics department. He then designed a combined coronagraph and prismatic camera of 40 feet focal length, which was constructed for use in Spain during the solar eclipse of 1905, but he was prevented by illness from accompanying the expedition. His place was taken by Dr E. H. Rayner, but no observations were possible on account of clouds.

In 1905 Mr Shackleton was appointed an Inspector of Scientific Supplies at the India Stores Depot, and retained this post to the time of his death. His training and experience had given him excellent qualifications for the duties of this office, and he introduced several improved methods of testing optical instruments, some of which have been described in the *Transactions* of the Society. His last communication to the Society, on "The Testing of Heliograph Mirrors," was presented shortly before his death and he was unable to read the proofs. Scientific workers in India have expressed in very generous terms their appreciation of Shackleton's careful supervision of the instruments and accessories sent out for their use. His work naturally brought him into close association with manufacturers, and it is pleasant to record that his relations with them were always most cordial.

For some years past Mr Shackleton had suffered considerably from bronchitis and kindred ailments, but his death, on June 26th, came unexpectedly. He was laid to rest at Putney Vale Cemetery in the presence of many relatives and friends, including representatives of the Optical and Royal Astronomical Societies. He leaves a widow and one daughter.

A. F.

REVIEWS

Penrose's Annual: The Process Year Book. Vol. 24. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S. Pp. xii + 91. (London: Lund, Humphries & Co., Ltd.) Price 8s. net.

The new volume of *Penrose's Annual* which has just been published maintains the high standard of the two previous volumes which have been reviewed in the *Transactions*. It contains the following articles by the Editor and other experts in process work and kindred subjects: "A Review of Process Work," "The Work of the Private Printing Presses," "High-light Negatives from Pencil Drawing," "Aerial Photography," "A Method of improving Reproduction by the Three-Colour Process," "Fashion Journals Old and New," "Industrial Research," "The 'New Method' Embossing," "Printers' Metals," "The Airbrush and Lithography," "The Print Society," "Tests of Acid Restrainers in Iron Developer for Wet Plates," "Engraving an Embossing Die from Photographs," "Lithotex Jobs," "The Drying of Printing Inks," "Stock-tints: Why not use them more?" "Screen Separations," "Mounting: A Standard Operation," "Theoretical Overlaying," "The Munsell Colour System," "Drying Printed Matter by Electricity," "Progressive Process Lithography," "A New Method of making Colour Prints," "A New Three-Colour Camera," and "A Revolution in Contents Posters." In addition to a number of plates illustrating some of these articles there are over forty beautiful illustrations representing process work during the year 1921. Besides being highly instructive, the book gives one a very good idea of the high standard to which process work in this country has attained. An additional incentive to purchase the book may be found in the fact that its price is lower than those of previous volumes.

J. S. A.

The Adjustment and Testing of Telescope Objectives. Third edition. Pp. 123; 30 figs. and 3 plates. (London and York: T. Cooke and Sons, Ltd., 1921.) Price 12s. net.

The optical world will welcome the appearance of a third edition of this very useful little book, as the previous edition has been out of print for some time. Although primarily intended for astronomers, the book is a source of valuable information to many other workers. The main portion of the book, although it deals very fully with the more important adjusting and testing methods, presumably those employed by Messrs Cooke, is not an exhaustive treatment of the subject, as no mention is made of other methods which are sometimes used, such as the Hartmann test. The book contains, in addition to the main parts designated by the title, reprints of papers by Mr H. Dennis Taylor on "The Secondary Colour Aberrations of the Refracting Telescope in Relation to Vision," "Description of a Perfectly Achromatic Refractor," and "An Experiment with a $1\frac{1}{2}$ -inch Refractor." In the present edition a chapter has been added on the adjustment of the Triple Photo-Visual Objective constructed by Messrs Cooke. The book can be highly recommended to all interested in the design, adjustment, testing, or use of telescope objectives.

J. S. A.

The Microscope: A Simple Handbook. By CONRAD BECK. Pp. 144; 131 figs. (London: R. and J. Beck, Ltd., 1921.) Price 2s. 6d. net.

This little book is a compendium of useful information for those about to take up, or who have begun to take up, the microscope for any of its many and ever-increasing practical applications, or for recreational purposes. It aims chiefly at teaching the novice how to select the most suitable apparatus for his needs, and how to manipulate his instrument to obtain the best results, by explaining the functions of the component parts of the microscope and its various appliances, without, however, trenching more on optical theory than the average user, who may not be well versed in optics, can easily grasp.

To experienced microscope users the interest of the book is perhaps restricted somewhat by the fact that it refers only to the products of one firm, but these also will find a number of handy tables in the book, such as those relating to field of view with various combinations of objectives and eyepieces, average alterations of draw tube lengths for different cover glass thicknesses, etc., as well as concise descriptions and explanations of many comparatively new and unfamiliar pieces of apparatus of considerable value. Amongst these may be mentioned a double wedge light moderator; a high power dark ground illuminator with an ingenious arrangement for focussing through slips of various thicknesses; a novel form of vernier micrometer eyepiece, invented by Mr Beck; a simple micro-spectroscope; simplified arrangements for making blood counts; and new forms of vertical illuminators not subject to the inconvenience of getting out of adjustment when focussing the microscope.

A neat little optical point, which we do not remember seeing in other microscope text books, is the explanation given by the author to show that although for measurement purposes the microscope image is always assumed to be seen at 10" distance as the size of the image seen depends on the retinal area it occupies, it makes no difference whether the eye focusses the image nearer or farther away.

The author's views on the advantages of convergent as compared with parallel tubes in the use of binocular microscopes are interesting and merit attention.

We find here and there a few statements to which all microscopists would not subscribe, and on p. 43, where it is stated that visual resolution can be increased about 25 per cent. by the use of "purple" light, this is surely a misprint for "violet."

On the whole, however, this little book is singularly free from misprints or contentious matter, and is commendable for its simple and lucid style, and the convenient way it is got up for reference purposes.

J. R.

NOTICE

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